

Reply to S.J. Cox and D. Weaire

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(received ; accepted)

PACS. 82.70Rr – Aerosols and foams.

PACS. 47.60+i – Flows in ducts, channels, nozzles, and conduits.

PACS. 47.55Mh – Flows through porous media.

Cox and Weaire [2] rightly emphasize that our solution of the drainage equation for the Eiffel Tower geometry does not properly treat the boundary conditions. There should be a no-flow condition at the top, and, after leakage begins, the liquid fraction should be pegged to $\varepsilon_c \approx 0.36$ at the bottom. They then show how no-flow conditions at the top can improve agreement with numerical solution. But as argued in [1], we maintain that the neglect of boundary conditions is tantamount to neglect of capillary effects and that this cannot explain our measurements. At short times, capillarity can delay the onset of leakage, and at long times it can counter gravity and retain liquid in the foam indefinitely; in either case, leakage is *slower* than our approximate solution, contrary to experiment. Therefore, we speculated that the discrepancy arose from neglect of coarsening, whereby the average bubble size increases via gas diffusion from smaller to larger bubbles. This is an important puzzle because, while the drainage equation successfully predicts forced-drainage experiments, it fails dramatically for free-drainage experiments [1].

While [2] implements boundary conditions at the top of the sample, we clarify here the role of conditions at both boundaries. But first we remark that the no-flow condition obeyed by Eq. (4) of [2] was achieved only by introducing other approximations. As Cox and Weaire note, their solution does not satisfy the full drainage equation between the top and a depth that grows with time (eventually extending to the whole sample). Furthermore, it forces the liquid fraction to vanish at the top, $\varepsilon(0, t) = 0$, which is unphysical. Thus Eq. (4) of [2] need not necessarily have improved upon our original approximation.

In Fig. 1 we present a series of numerical solutions. In the notation of [1], the sample height is $H = 70$ cm, the flaring length is $z_o = 25$ cm, the capillary rise scale is $\xi = 5$ cm, the drainage time scale is $t_o = 2600$ s, and the initial liquid fraction is $\varepsilon_o = 0.36$. We also take $m = 1$ as in [2]. Our predictions [1] are that the liquid fraction profile is $\varepsilon(z, t) = \varepsilon_o / (1 + t/t_o)$ and that the normalized volume of drained liquid is $V(t)/V_f = 1/(1 + t_o/t)$; i.e. the drainage is uniform and all the liquid eventually leaks out. By contrast the full numerical solution for $\varepsilon(z, t)$, in Fig. 1a, shows that the sample becomes drier at the top and remains wet at the bottom. Consequently, in Fig. 1b, $V(t)/V_f$ does not approach 1 at long times. Also in Fig. 1a-b, we see that the liquid fraction at the top, $\varepsilon(0, t)$, is not zero as assumed in [2].

Next we disable the boundary conditions, one at a time, by taking $\partial\varepsilon/\partial z = 0$ as in our approximate solution. When only the bottom boundary conditions are correct, we find the

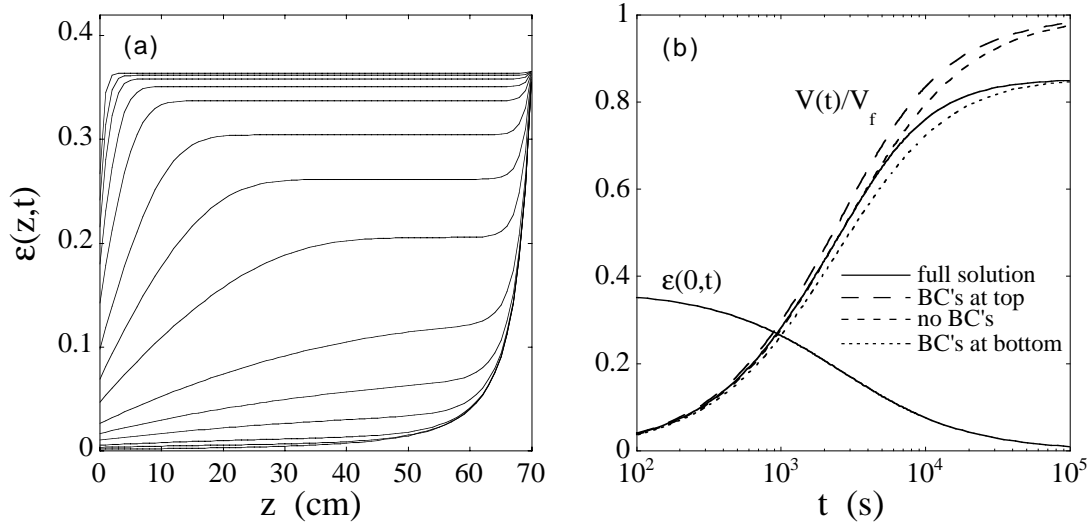


Fig. 1. – (a) Liquid fraction vs depth in a draining foam. From top to bottom the times shown follow a 1-2-5-10 sequence from 10 s to 10^5 s; the very bottom curve is the equilibrium profile. (b) Volume of drained liquid vs time for various combinations of boundary conditions, as labeled. The liquid fraction at the top of the sample $\varepsilon(0, t)$ is also shown.

dotted curve of Fig. 1b, in which the volume of drained liquid is correct at short times and at long times. The final approach is too slow because of “flow” conditions at the top, as though liquid were entering the sample from above. When only the top boundary conditions are correct, we find the long-dashed curve of Fig. 1b, in which the volume of drained liquid is correct at short times. For intermediate and late times the drainage is too fast, and all the liquid eventually leaves the sample, because capillary forces no longer act at the bottom boundary. When neither boundary condition is correct, we have our original approximation, the short-dashed curve of Fig. 1b. Since the errors introduced at top and bottom act in opposite directions, there is some cancellation and hence the agreement with the full numerical prediction extends to later times than when just one of the boundary conditions is disabled. However, at the latest times, the error due to neglect of the bottom boundary conditions dominates and all the liquid leaves the sample.

Our conclusions thus differ somewhat from those of [2]. We agree that the bottom boundary conditions are the most important to implement correctly. But we believe that the top boundary conditions can be neglected to good approximation at both short and long times. If the bottom boundary conditions must be dropped, then the top boundary conditions should be dropped as well to maintain better accuracy at intermediate times.

REFERENCES

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- [2] COX S.J, and WEAIRE D, *Europhys. Lett.*, (2001) preprint.