Cooperativity

- Phase Transformations:
  - many different kinds of phase transitions: dimension, microscopic origin…
  - cooperativity (dominos' effect) play a key role

Example: para-ferromagnetic transition

- Magnetic dipoles partially align in an external magnetic field
- An aligned dipole generates additional field which adds to the external fiel:

$$\vec{B}_{loc} = \vec{B}_{ext} + \vec{B}_{ind}$$

- Linear response: magnetization proportional to local field

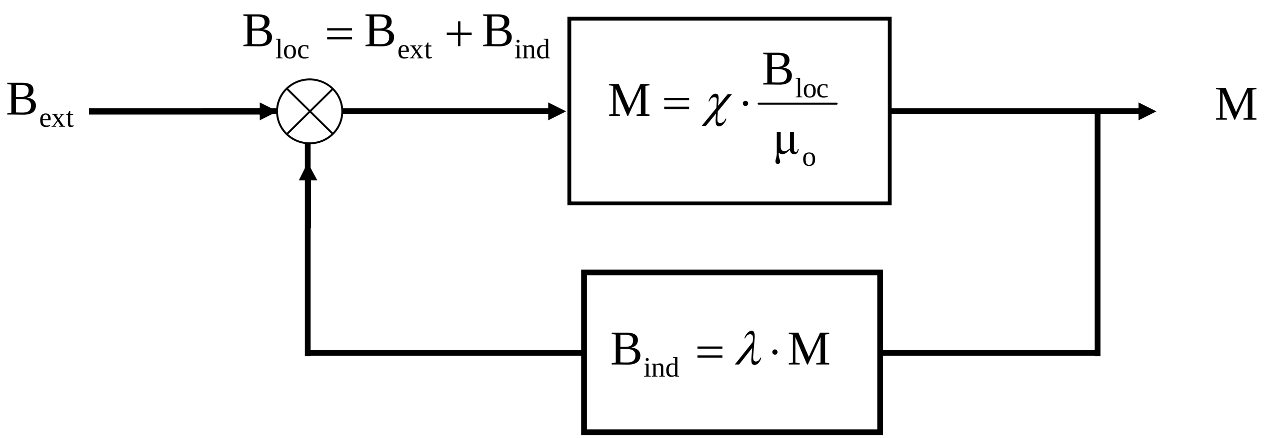
$$\vec{M} = \chi \cdot \frac{\vec{B}_{loc}}{\mu_o}$$

Cooperativity

- Phase Transformations:

Example: para-ferromagnetic transition

- Linear response: magnetization proportional to local field



$$\chi_{eff} = \frac{M}{B_{ext}} = \frac{\chi/\mu_o}{1 - \lambda\chi/\mu_o}$$

M goes to infinity (divergence) if $\lambda\chi/\mu_o \rightarrow 1$

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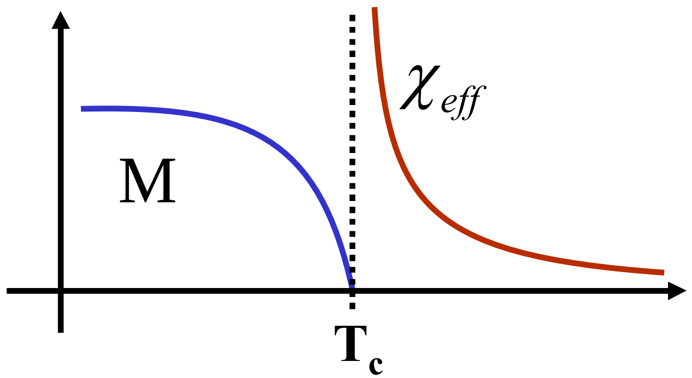
  - divergence of susceptibility at T_c:



- below T_c, ordering of dipoles exist "spontaneously" (without external field) preventing from infinite magnetization and infinite magnetic energy storage in the presence of an external field

- spontaneous magnetization in the low temperature phase is called an **order parameter**. *In the high temperature, the order parameter is zero*.

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- order parameter fluctuations :

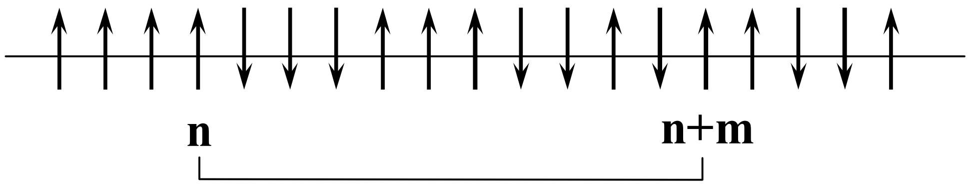
▪ **above Tc :** two dipoles at infinity from each other are uncorrelated, but for short or mean range, correlation between dipoles may exist "locally" in space and time and the net macroscopic magnetization still zero.

$$\langle \mu_n \mu_{n+m} \rangle = 0 \qquad \text{if} \quad n \neq m$$

▪ **below Tc :** correlation (or coherence) length of ordered phase is infinite and the net macroscopic magnetization is non zero.

$$\langle \mu_n \mu_{n+m} \rangle \neq 0$$

pair correlation function

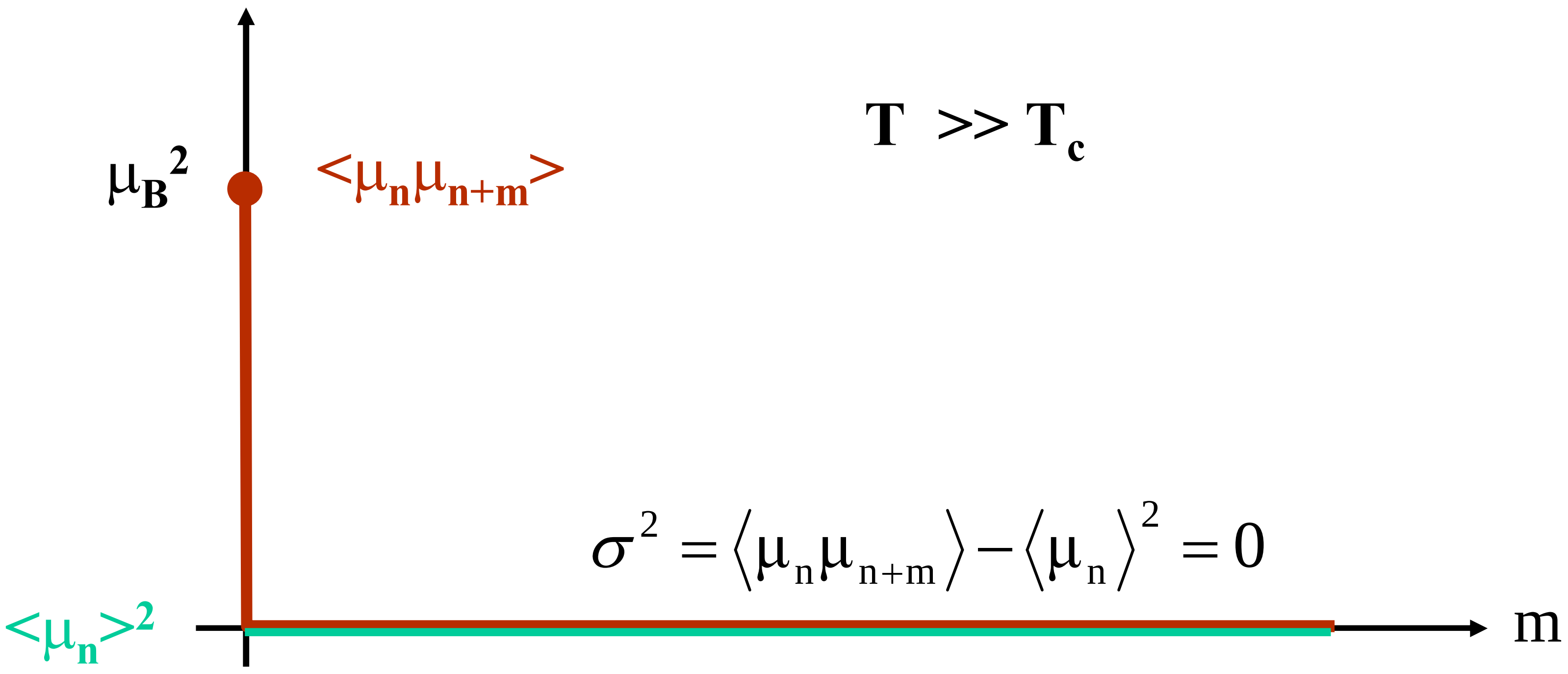


Cooperativity

- Phase Transformations:

Example: para-ferromagnetic transition

- correlation length : the origin of “n” site doesn’t matter (ergodicity)



$$\sigma^2 = \langle \mu_n \mu_{n+m} \rangle - \langle \mu_n \rangle^2 = 0$$

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- Phase Transformations:

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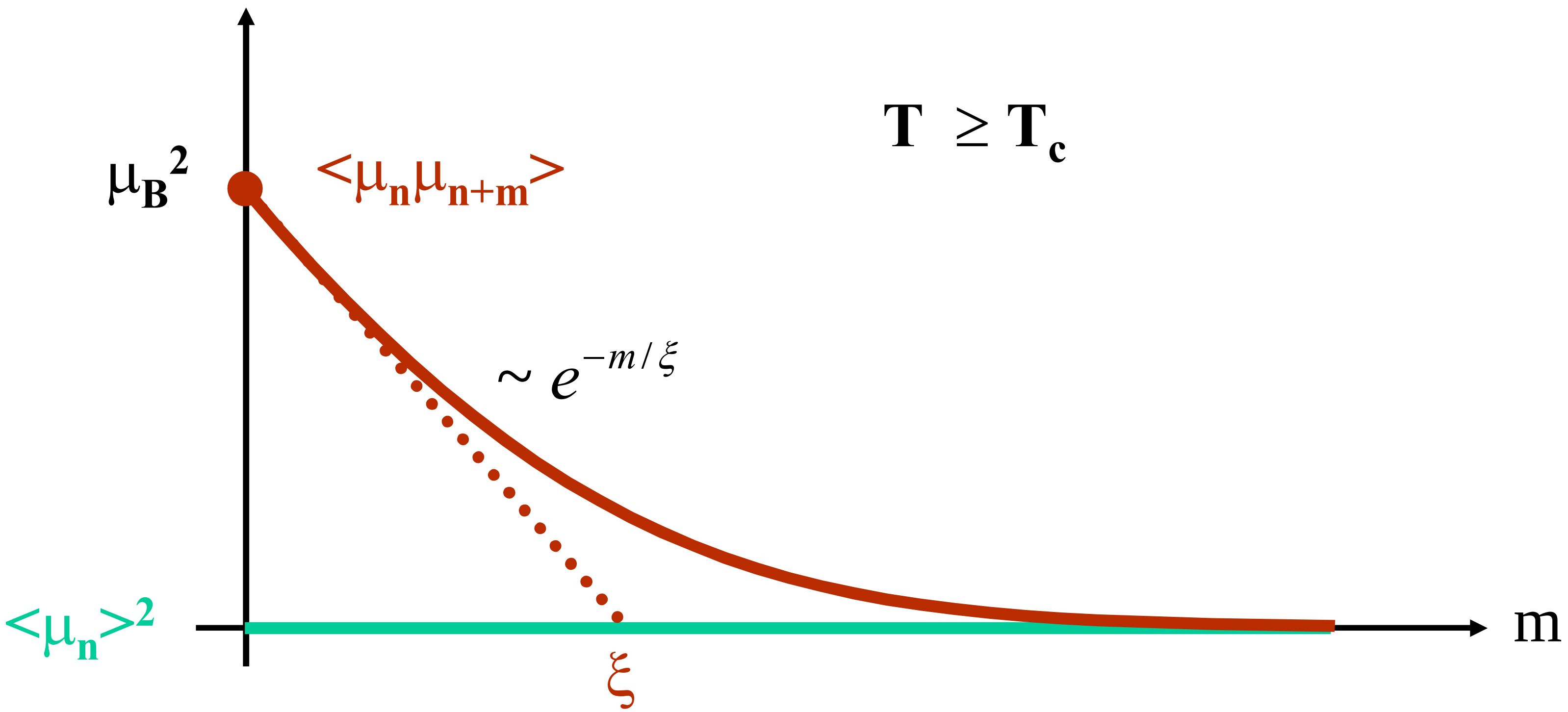
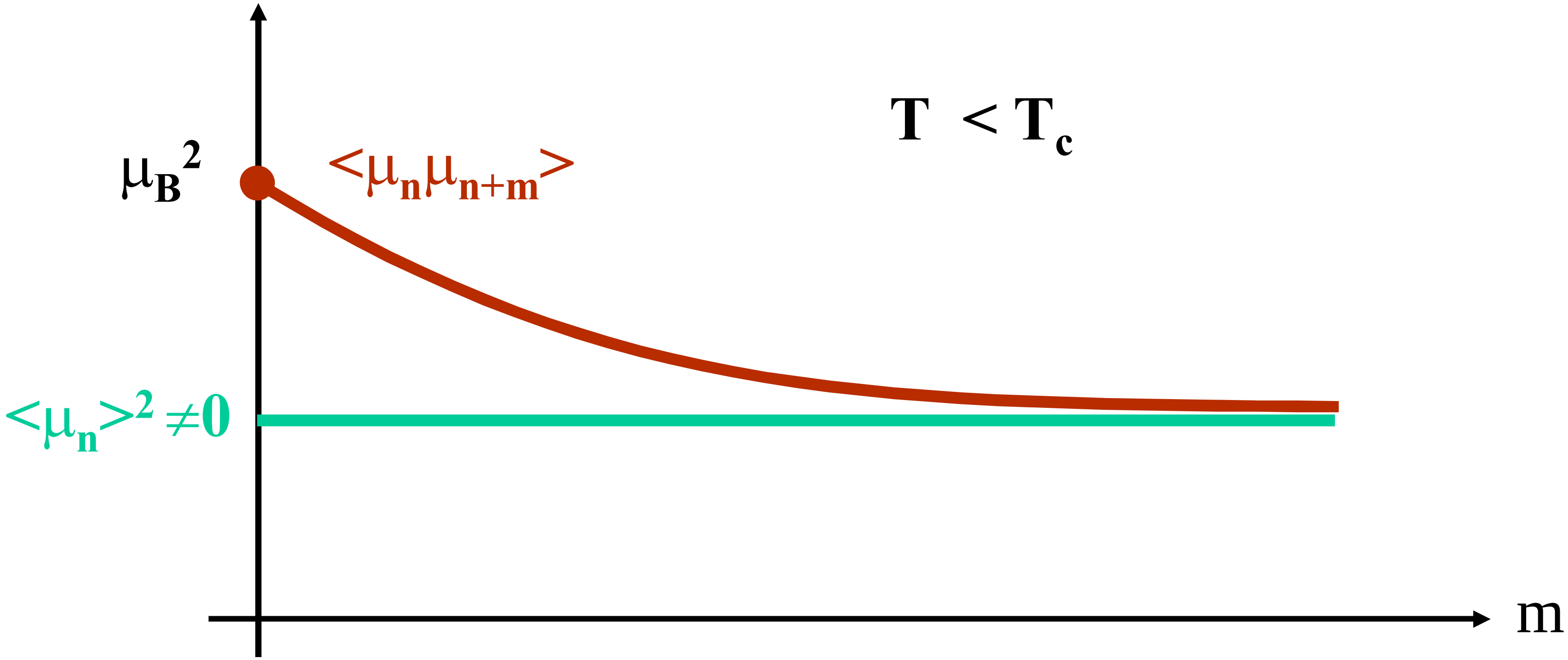


Cooperativity

- Phase Transformations:

Example: para-ferromagnetic transition



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  Example: para-ferromagnetic transition