

**"Algebra Geometry and Algorithms for young mathematicians in Africa"**  
**"Algebre Geometrie et Algorithmes pour les jeunes mathematiens africains".**  
**Location/lieu Ecole des Sciences, Departement de Mathematiques**  
**Universite Abdou Moumouni, Niamey Niger**  
**du 21 au 25 février 2005**

Monday 21/Lundi 21

8H 9H 30 Welcome /Accueil

9H 30 10 H Opening ceremony/Ceremonie d'ouverture

10H 30 12 H

Daouda Sangare (Cote d'ivoire)

On the Asymptotic behaviour of the Hilbert function of the fiber cone of current filtrations and an extension of the concept of analytic spread via the degree function.

12H 15 H Lunch and pause

15H 16H 30

Augustin Banyaga (USA)

The geometry of the group of symplectomorphisms.

17H 18 H 30

Rikard Bøgvad (Suede)

D-modules and computing with them.

Tuesday 22/Mardi 22

8H 30 10 H

Francesca Acquistapace (Italie)

Real analytic normal surfaces

10H 30 12 H

Marie-Francoise Roy (France)

Discriminants of symmetric matrices are sums of squares

12H 15 H Lunch and pause

13H 30 15 H Meeting of RAGAAD coordinators

15H 16H 30

Henri Lombardi (France)

Constructive theory of dimension

17H 18 H 30

Omar Sall (Dakar)

Algebraic geometry on finite fields

Wednesday 23/ Mercredi 23

8H 30 10 H

Momo Bangoura (Guinee)

Equation de Yang-Baxter classique et groupes de Lie-Poisson .

10H 30 12 H

Théodore Tapsoba (Burkina)

Combinatoire des mots.

12H 15 H Lunch

Free afternoon

Thursday 24 /Jeudi 24

8H 30 10 H

Aviva Szpirglas (France)

Toric geometry

10H 30 12 H

Garba Bellko (Niger)

Analytic geometry

12H 15 H Lunch and pause

15H 16H 30

Michel Coste (France)

Quantitative results on semi-algebraic sets

17H 18 H 30

Jounaidi Abdeljaoued (Tunisie)

Algorithmes et Complexité en Algèbre linéaire

Friday 25/Mardi 25

8H 30 10 H

Mahaman Bazanfare (Niger)

Riemannian geometry: comparison theorems.

10H 30 12 H

Issoufou Katambe (Niger)

Genetic algebra

12H 15 H Lunch and pause

13H 30 15 H Meeting of RAGAAD coordinators

15H 16H 30

Fabrizio Broglia (Italie)

Real analytic geometry

17H Closing ceremony

19 H 30 Final conference dinner

## Résumé des exposés Summary of lectures

*Jounaidi Abdeljaoued* (Tunisie)

Algorithmes et Complexité en Algèbre linéaire

Après un bref rappel des principaux outils d'Algèbre linéaire utilisés, et une introduction aux modèles de calcul adoptés, un aperçu sera donné sur les algorithmes déterministes les plus performants pour le calcul sans divisions du polynôme caractéristique d'une matrice carrée à coefficients dans un anneau commutatif arbitraire. Il s'agit notamment de l'algorithme amélioré de Samuelson-Berkowitz, de l'algorithme de Chistov et de l'algorithme de Kaltofen-Wiedemann, ce dernier utilisant la théorie des suites récurrentes linéaires et le procédé d'élimination des divisions dû à Strassen. On s'intéressera plus particulièrement dans cette première partie aux méthodes correspondant à des circuits arithmétiques de faible profondeur et, dans le cas de  $\mathbb{Z}$ , à leur complexité binaire.

La deuxième partie de l'exposé sera consacrée à la présentation d'un travail commun avec G. Malaschonok sur les méthodes quasi-triangulaires pour le calcul du polynôme caractéristique sur un anneau commutatif avec division exacte, et d'un deuxième résultat plus récent obtenu avec G. Diaz-Toca et L. Gonzalez-Vega sur les mineurs caractéristiques de Bézout, comme application du calcul rapide du polynôme caractéristique à la paramétrisation du PGCD de deux polynômes à une indéterminée dont les coefficients dépendent de plusieurs paramètres décrivant un corps réel clos.

*Francesca Acquistapace* (Université de Pisa, Italie)

Real analytic normal surfaces

After a short motivation based on Hilbert 17 Problem, I will introduce the set of orderings on a field. In particular we shall see how associate to any ordering on the field of meromorphic functions a filter of global semianalytic sets which enables to find conditions for a function to be positive with respect to an ordering.

After this I will present the paper "The Artin-Lang property for normal real analytic surfaces." J. Reine Angew. Math. 556 (2003), 99--111. by C.Andradas, A.Diaz-Cano, J.Ruiz where the authors solve Hilbert 17, prove Artin Lang property and characterize the absolute and the relative holomorphy rings of the surface.

*Augustin Banyaga* (USA)

The geometry of the group of symplectomorphisms.

The first part of this talk is devoted to the basics (symplectic manifolds, symplectic and Hamiltonian diffeomorphisms, Lagrangian submanifolds, etc...) and to review the main breakthroughs in Symplectic Topology and Geometry of the last 3 decades (including the Arnold conjecture Gromov's J holomorphic curves and Floer homology).

In the second part, I will introduce the Hofer Geometry and discuss the Hofer norm: a bi-invariant metric on the group of Hamiltonian diffeomorphisms and outline the recent work by Banyaga-Donato extending the Hofer norm to the group of all symplectomorphisms of certain symplectic manifolds and an extension to the group of strictly contact diffeomorphisms of a regular contact manifold.

Lecture notes and relevant papers will be distributed.

*Rikard Bøgvad* (Suède)

D-modules and computing with them.

The ring of differential operators on a smooth variety and its modules are as natural an invariant of the variety as the ring of functions. They encode topological information of the variety, as well as are interesting in their own right as representing certain differential equations, that occur in the very origins of algebraic geometry. I will try to motivate the theory, introduce certain classes of especially interesting modules and then describe papers by Oaku and Walther which describes how to construct algorithms for computing e.g. decomposition under localization and the b-function.

*Fabrizio Broglia* (Université de Pisa, Italie)

Real analytic geometry

Some standard problems for semialgebraic sets are presented in the semianalytic framing. After a short overview on the state of the art I will present the paper "Connected components of global semianalytic subsets of 2-dimensional analytic manifolds" (by Anna Castilla and Carlos Andradas; J. Reine Angew. Math. 475 (1996), 137—148) where it is proved that the any connected component of a 2 dimensional global semianalytic is still global semianalytic. The result is the most advanced on this subject: the problem for general dimension is still open.

*Michel Coste* (Université de Rennes I, France)

Quantitative results on semi-algebraic sets

Je prévois de donner quelques idées générales sur les bornes effectives que l'on peut obtenir pour certains invariants métriques d'ensembles décrits par des équations et inégalités polynomiales, en fonction du nombre de variables et du degré des polynômes intervenant dans la description. Je présenterai surtout les récents résultats de Dider d'Acunto et Krzysztof Kurdyka concernant le diamètre géodésique: on borne la distance qu'il faut parcourir pour aller d'un point à un autre d'un ensemble connexe en restant dans cet ensemble.

*Marie-Françoise Roy* (Université de Rennes I, France)

Discriminants of symmetric matrices are sums of squares

In this first part I shall discuss the notions and properties of discriminants and subdiscriminants and in the second part I shall prove that considered as a polynomial in the entries of a symmetric matrix, the subdiscriminants are sums of squares. My lecture will be based on the preprint « On some identities for the elements of a symmetric matrix' » by N.V. Ilyushechkin and some short notes I wrote recently.

*Daouda Sangare* (Cote d'Ivoire)

On the Asymptotic behaviour of the Hilbert function of the fiber cone of current filtrations and an extension of the concept of analytic spread via the degree function.

We shall recall classical definitions and results of [RS] and extend them to a more general class of filtrations. This lecture contains some results published previously, jointly or not, by the following

authors who belong to the Team of Commutative Algebra of the University of Abobo- Adjamé (Abidjan, Côte d'Ivoire), the Ecole Normale Supérieure of Abidjan (Côte d'Ivoire) and the University of Clermont - Ferrand (France) : \ D. Sangaré, Ph. Ayégnon, Y. Diagana, A. Assane and H. Dichi. Many documents (papers and books) will be available for interested students in the area of numerical polynomials, functions of polynomial type, quasi-polynomial functions, Hilbert functions etc..

## References

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Abdoulaye Assane, Sur le comportement asymptotique de la fonction de Hilbert du cône fibré d'une filtration fortement noethérienne et la conjecture de Okon, }Afrika Matematika, Serie 3, vol.13 (2002), 35 - 51.

A. Assane, P. Ayégnon et D. Sangaré, On the asymptotic nature of the analytic spread of I-good and strongly noetherian filtrations, Afrika Matematika, Serie 3, vol.12 (2001), 51-60.

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W. Bishop, J.W.Petro, J. Ratliff .Jr., D.E. Rush, Note on noetherian filtrations, Comm. Algebra. 17 (2) 471 - 485 (1989)

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Y. Diagana, H. Dichi and D. Sangaré, Filtrations, Generalized analytic independence, analytic spread, Afrika Matematika, series 3, vol.4 (1994)

H. Dichi, D. Sangaré, Hilbert functions, Hilbert-Samuel quasi-polynomials with respect to f-good filtrations, multiplicities, J. Pure Applied Algebra, 138, ( 1999) 205 -213 [DS2]

H. Dichi, D. Sangaré, Analytic spread of filtrations, asymptotic nature and some stability properties, Comm. Algebra, 28 ( 7 ), 3115 - 3124 (2000)

G. Northcott, D. Rees, Reduction of ideals in a local ring, Proc. Camb. Philos. Soc., 50 ( 1954 ), 145 - 158

J. S. Okon, Prime divisors, analytic spread and filtrations, Pacific J. Math., 113, 2, ( 1984 ) 451 - 462

D.Rees, Lectures on the asymptotic theory of ideals, London Math. Soc. Lecture Note Series 113, (1988)

*Aviva Szpirglas* (Université de Poitiers, France)

## Toric geometry

We will give the construction of toric varieties which generalizes the quotient construction of the projective space. We will first give a quick view on affine toric varieties, fans and toric varieties. We will then introduce the ring of homogenous coordinates for a toric variety, with several examples. The grading that can be attached to this ring can then be described easily. We will see how in this framework can be described some properties of toric varieties (as toric Nullstellensatz, or properties of divisors). We will then see how a Laurent polynomial with  $n$  indeterminates is "multihomogenized" and associated to a multihomogenous polynomial of the ring of homogenous coordinates of the toric variety associated to its Newton polytope.

We will give examples to illustrate the talk.