EUROPHYSICS LETTERS

OFFPRINT

Vol. 65 • Number 5 • pp. 726–732

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24 issues - Vol. 65-68 (6 issues per vol.)

ISSN: 0295-5075 - ISSN electronic: 1286-4854

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Europhys. Lett., **65** (5), pp. 726–732 (2004)
DOI: 10.1209/epl/i2003-10169-0

Dissipation in foam flowing through narrow channels

I. Cantat[1], N. Kern[2] and R. Delannay[1]

[1] *GMCM UMR 6626, Université Rennes I - 35042 Rennes cedex, France*
[2] *Physics Department, Trinity College Dublin - Dublin 2, Ireland*

(received 31 March 2003; accepted in final form 17 December 2003)

PACS. 83.50.Ha – Flow in channels.
PACS. 83.80.Iz – Emulsions and foams.
PACS. 83.85.Jn – Viscosity measurements.

Abstract. – We present measurements on simple foam structures flowing through a narrow channel. Driving pressure and bubble velocity are found to be related by a power law with exponent 2/3, thus extending this relationship from single bubble trains (see Bretherton F. P., *J. Fluid Mech.*, **10** (1961) 166) to more complicated foam structures. This nonlinear behaviour is due to the dominant dissipation mechanism, related to the sliding of Plateau borders over the channel walls. For the first time, we show that the prefactor in the power law strongly depends on the foam structure, through the orientation of the Plateau borders with respect to the flow direction. Our evidence suggests that the normal motion largely dominates the dissipation. We discuss implications for quasi-planar rheological experiments on foams confined between glass plates.

Introduction. – What pressure gradient is required to drive a foam through a capillary, a pore or a narrow channel? This is one of the simplest questions one can ask in the quest of understanding the rheology of foams. Nevertheless, even for the simplest case, a single bubble advancing through a circular capillary, theoretical work has shown that the process involves rather subtle hydrodynamics, which lead to a nonlinear relation between the required pressure and the desired flow velocity for the bubbles [1]. Historically, this question has been raised by the need to pump foam through a porous medium, *e.g.* in the context of oil recovery, with a typical pore size of the order of microns. Here, our motivation stems from the recently revived interest in experiments on foam rheology in a quasi–two-dimensional geometry (see, *e.g.*, [2–4]), or in a classical rheometer for which sliding effects on the plates may be crucial. In the "2D" experiments a foam is confined between two parallel glass plates, which are separated by a distance smaller than the bubble radius (typically by a few millimeters). In many respects, this system can be regarded as a two-dimensional equivalent of a foam, the bubbles being characterized by their projected area (rather than their volume) and subject to an effective line tension (rather than a surface tension). However, a signature of the third dimension persists by way of the liquid channels, located between neighboring bubbles and the glass plates (see fig. 1a). When the whole foam is pushed ahead between the plates, complex flows take place in the network of these "Plateau borders" as they slide on the plates. Most of the injected energy is dissipated in this process [5], which involves nonlinearities arising from the deformability of the films delimiting the Plateau borders. In the past, several groups have carried out theoretical, numerical and experimental work on the problem of one bubble or

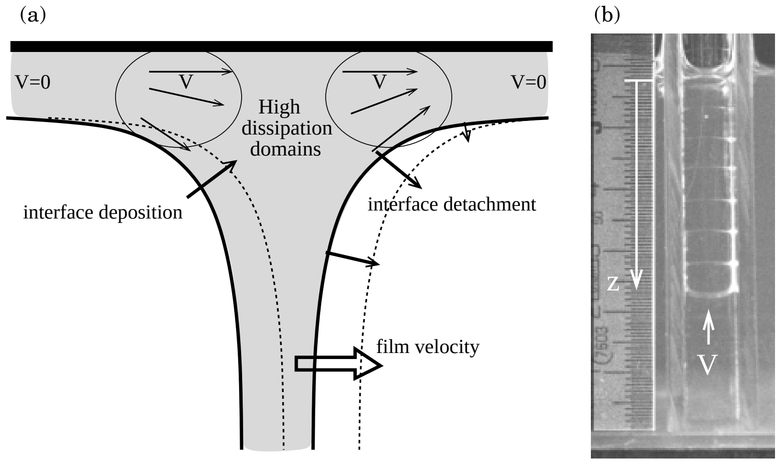

Fig. 1 – (a) Fluid flow around a Plateau border touching the wall. The wetting film is almost at rest on the channel wall and dissipation is dominated by the local flow required to depose the film onto the capillary at the front of the bubble and to detach it at the back. (b) Experimental setup, case of a "1B" structure.

of a line of bubbles (called "1B") advancing through a cylindrical or polygonal capillary. A scaling relation for the pressure drop in relation to the flow velocity V of the bubbles

$$\Delta p \sim Ca^{\alpha} \sim V^{\alpha} \qquad \text{with} \qquad \alpha = \frac{2}{3} \tag{1}$$

has been found in the regime of small capillary numbers ($Ca = \mu V/\sigma \ll 1$, where μ is the viscosity of the liquid phase and σ the surface tension at the interface) [1,5–8].

In the present study, we determine experimentally the pressure-velocity relation for a more complex foam structure advancing through a rectangular channel, thus identifying the dominating dissipation mechanism relevant for quasi-planar rheological experiments. We check the power law (eq. (1)) and we discuss the prefactor, showing that it is strongly affected by the orientation of the Plateau borders with respect to the velocity direction. Our model, which disregards the dissipation associated with their tangential motion, leads to good agreement with the experimental results.

Experimental setup and procedure. – As foaming agent, we use a solution of commercial dish-washing fluid (10% in volume), glycerol (5% in volume) and water. The viscosity is $\mu = 1.16 \cdot 10^{-3}\,\mathrm{N\,s\,m^{-2}}$ and the surface tension $\sigma = 27 \cdot 10^{-3}\,\mathrm{N\,m^{-1}}$, as already measured in [9]. The foam is produced in a small Plexiglas channel (cross-section $S = a_1 \times a_2 = 3\,\mathrm{mm} \times 9\,\mathrm{mm}$, length $a_3 = 200\,\mathrm{mm}$) by blowing N_2, which leads to well-controlled regular foams, as in [10]. By varying the typical bubble size (via gas flux and pressure) we obtain various structures with only one layer of bubbles in the smallest dimension a_1, and between one and three layers of bubbles in the dimension a_2, denoted, respectively, by "1B", "2B" and "3B" structures (see fig. 1b and 2a, b). The entire foam involves a total of 20 to 50 bubbles. Taking advantage of the very small polydispersity in bubble size (less than 5%) and of the perfect regularity of the structure, it is sufficient to perform detailed geometrical measures (see below) on one unit cell only, using images recorded by a camera oriented in the direction a_1 (see fig. 2a). Flow is along the channel (direction a_3); enough free space remains at the end of the column for foam

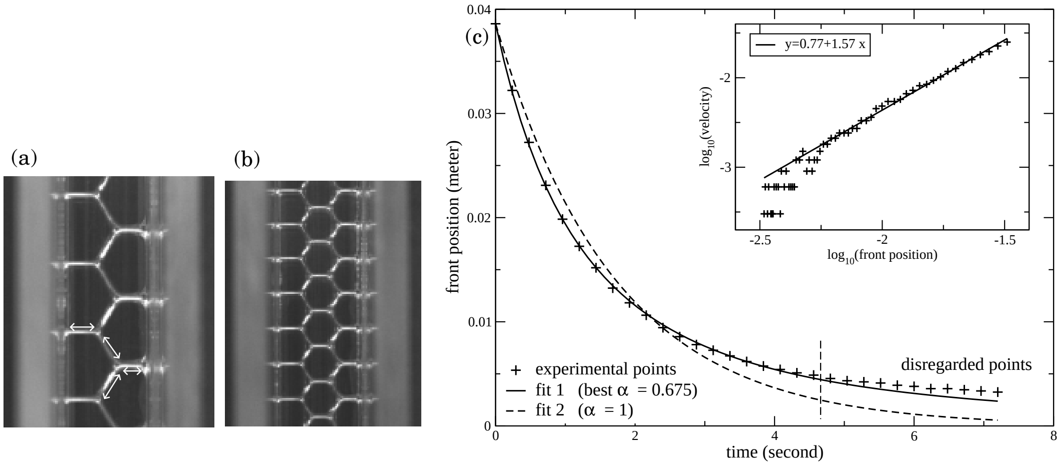

Fig. 2 – (a) "2B" structure (L_{wet} is indicated for one periodic cell), (b) "3B" structure, (c) velocity power law. Evolution of the interface as a function of time, for one data set, indicating a power law with exponent $\alpha \approx 2/3$. See the text for a discussion of the cutoff.

to advance through the channel without bubbles leaving it.

Once prepared, the foam is kept at rest for 15 minutes, the direction a_1 being vertical. For waiting times varying between 10 and 30 minutes, experimental results were unchanged. The whole channel is then quickly submerged vertically into a bowl containing typically 10 cm of the foaming solution, reaching the bottom at time $t = 0$. Pushed by the fluid, the foam rises in the channel at a velocity V of the order of a few cm s^{-1} (see fig. 1b).

Pressure forces. – At this velocity scale ($Ca \simeq 10^{-4}$–10^{-3}), the pressure distribution in the homogeneous fluid phase is hydrostatic and the driving force acting on the foam is simply

$$\boldsymbol{F}_{\text{pres}} = -z\rho g S \boldsymbol{e}_z, \tag{2}$$

where the immersion depth z is the height of the free water surface in the container with respect to the lowest air/water interface in the structure. $\rho = 10^3\,\text{kg}\,\text{m}^{-3}$ is the fluid density, $g = 9.81\,\text{m}\,\text{s}^{-2}$ the gravitational acceleration and $\boldsymbol{e}_z$ the vertical unit vector oriented downward. We record the motion of the foam with a CCD camera taking 25 images per second, interfaced with the software Visilog which we use to determine the position z on each snapshot. For different series of each type of structure, the bubble diameter may vary approximately from 3 mm to 1 cm (except for the "3B" structure, which is only stable in a small range of bubble sizes), the height of the foam column in the channel varies between 10 cm and 20 cm and the initial immersion depth between 5 cm and 10 cm.

Liquid fraction and drainage. – Our setup for producing the foam does not give us a fine control over the liquid fraction (reflected in the size of the Plateau borders): different settings for gas pressure, gas flow rate and waiting time may lead to variations in the liquid content, in particular when comparing samples with different structures. It would clearly be an improvement to be able to set the liquid fraction at will, and to measure dynamically the size of the Plateau borders and the thickness of the wetting films on the channel walls. However, theoretical predictions suggest that the liquid fraction does not affect the exponent in the pressure-velocity law, over a large range of the liquid fraction [6, 11]. Note that this is also essential for an interpretation of our data in terms of the aforementioned models, which are all formulated assuming a stationary regime: given the bubble velocity, its shape adjusts to balance mechanical forces, subject to the accompanying hydrodynamics in the liquid phase. The latter thereby defines the pressure drop across the bubble. Whereas the shape and size of the Plateau border implicitly selects the local liquid fraction as a function of velocity in such a steady-state model, the liquid fraction in a time-dependent experiment like ours may differ from this value, since the adjustment via drainage through the Plateau borders may be achieved only partially. None of this will affect the pressure-velocity scaling (1) if we assume that the effect of liquid fraction is negligible —an assumption which is *a posteriori* shown to be consistent once the scaling is validated.

Viscous forces. – As the foam rises, there is no relative motion between the bubbles (plug flow); dissipation occurs mainly in the Plateau borders touching the wall in a local process (cf. the introduction). Therefore, one expects the viscous force to be proportional to the wetting perimeter L_{wet}, *i.e.* the total length of Plateau border in contact with the boundaries (we defer the important discussion of the role of film orientation to the following section). Thus we write *a priori*

$$\boldsymbol{F}_{\mathrm{visc}} = \lambda n L_{\mathrm{wet}} \left(-\frac{\mathrm{d}z}{\mathrm{d}t} \right)^{\alpha} \boldsymbol{e}_z, \tag{3}$$

with $\boldsymbol{F}_{\mathrm{visc}}$ representing the total viscous force exerted by the wall on the foam, L_{wet} the total wetting perimeter of a periodic cell (without the vertices), n the number of periodic cells, $-\mathrm{d}z/\mathrm{d}t$ the (upward) bubble velocity, α the exponent to be determined and λ an unknown coefficient, the dimension of which depends on α.

In measuring L_{wet}, thought has to be given to the size of the vertices, where three Plateau borders meet. Theoretical models assume that the dominant dissipation is localized on the reconnection lines between Plateau borders and wetting films (see fig. 1a). Consistency therefore requires to reduce the wetting perimeter by the length corresponding to the finite size of the vertices (see fig. 2a). To estimate this correction from the digital picture, we estimate the pixel at which the thickness of the edge begins to increase, implying an error on the edge length measurement of the order of 5%. We find that this correction indeed improves the agreement between theory and experiment significantly.

Dynamics. – Since inertial forces ($\simeq 10^{-7}\,\mathrm{N}$) and gravitational forces ($\simeq 10^{-4}\,\mathrm{N}$) exerted on the foam are small compared to pressure and viscous forces ($\sim 10^{-2}$–$10^{-3}\,\mathrm{N}$), the latter have to cancel in order to achieve an instantaneous force balance:

$$\left(-\frac{\mathrm{d}z}{\mathrm{d}t} \right)^{\alpha} - z\frac{\rho g S}{\lambda n L_{\mathrm{wet}}} = 0. \tag{4}$$

For $\alpha \neq 1$, this leads to a trajectory given by

$$z(t) = \left(\beta K t + z_0^{-\beta}\right)^{-1/\beta} \qquad \text{with} \qquad \beta = \frac{1-\alpha}{\alpha}\,, \tag{5}$$

where z_0 is the interface location at $t = 0$ and $K = (\rho S g / n L_{\mathrm{wet}} \lambda)^{1/\alpha}$. This defines our model, which we use to interpret our experimental data. In the special case $\alpha = 1$, eq. (4) leads to exponential relaxation.

Before entering the details of this analysis, it is important to remark that the rise of the foam column stops relatively far below the free water surface (typically at $z = 5\,\mathrm{mm}$). Although the reasons for this are not entirely clear, our interpretation is that dewetting occurs

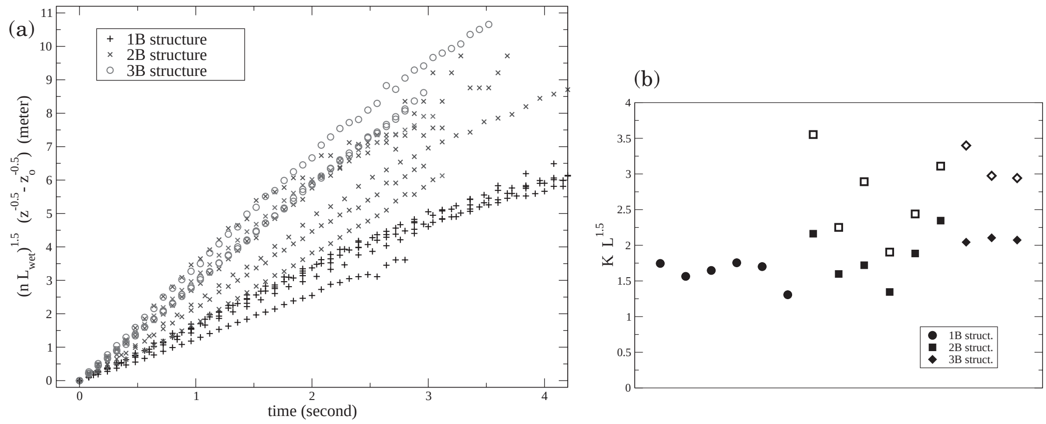

Fig. 3 – (a) A rescaled position variable, $nL_{\mathrm{wet}}^{3/2}(z^{-1/2} - z_0^{-1/2})$, assuming an exponent $\alpha \approx 2/3$, plotted as a function of time for various foam structures. The linearity is well respected, indicating that this exponent is indeed appropriate; deviations for long times are expected (see text). However, the data is split into families of curves corresponding to different structures; such a difference in slope points to an influence of the orientation of edges. (b) The slope is characterized by the parameter K (see eq. (5)), here rescaled by $L_{\mathrm{wet}}^{3/2}$ (empty symbols) and by $L_{\mathrm{proj}}^{3/2}$ (filled symbols), respectively, and shown for all samples (but note that $L_{\mathrm{wet}} \equiv L_{\mathrm{proj}}$ for the "1B" structure). Using the projected length improves the superposition of the various slopes.

when the motion slows down asymptotically, thus leading to pinning forces on the plexiglass, which we do not expect to arise at higher velocity when the wetting films are dynamically swollen. The induced capillary forces can attain the order of $\sigma \times$ perimeter $\approx 10^{-3}$ N, in good agreement with the remaining pressure force $P \sim 50$ Pa. For these reasons we introduce a cut-off, excluding front positions smaller than 5 mm depth. We do not shift the pressure according to this reference. The following simple test is worth noting: once at rest few millimeters above the water surface, the foam can be unpinned by inducing a small vertical oscillation of the channel. In this case the foam motion is initially forced, thus swelling the film; when the channel stops, the foam then keeps moving and finally reaches the position corresponding to the water level, as expected in the absence of pinning forces, and in good agreement with the previous interpretation of the high- and low-velocity behavior.

Experimental results: exponent and influence of film orientation. – A typical result of our raw experimental data, the position of the front of the foam as a function of time, is presented in fig. 2c. They closely follow the curve predicted by our model eq. (5), where the exponent $\alpha = 0.675$ has been obtained as best fit. For comparison, we also show the curve which would correspond to an exponential relaxation (exponent $\alpha = 1$). Similar curves are obtained for several samples of all three structures, and all our exponents are consistent with $\alpha = 2/3$. Figure 3a shows all experimental results, rescaled according to this power law ($\alpha = 2/3$). Linearity is well respected, showing that this exponent is indeed consistent. However, the slope is *not* universal. We shall show that this can be explained if the orientation of the Plateau borders with respect to the flow direction is taken into account.

To do so, we assume that dissipation arises mainly from the *normal* motion of the Plateau borders. That this is indeed a reasonable assumption may be inferred from the flow of bubbles through rectangular channels, where it has been shown that the (parallel) displacement of "corner" Plateau borders is indeed *not* the main dissipation mechanism [7]. The relevant length is therefore the projection of the wetting length onto the channel cross-section L_{proj}, rather than its total length L_{wet}. Note that the ratio of these two lengths varies from one type of structure to another, but also from one experimental series to the next, since the bubble size may have changed. Figure 3b shows that this rescaling indeed allows to strongly reduce the slope dispersion. We have not been able to correlate the remaining variation in the slope with any other experimental parameter. It may partly arise from a (small) polydispersity in bubble size, as the length measures are performed on a single periodic cell. Second-order contributions of the dissipation related to tangential motion and of the liquid fraction might also give rise to corrections to the dynamics too. Quantification of these contributions will require to decrease the intrinsic noise of the experimental setup.

For completeness and comparison, let us also discuss the numerical value of the prefactor. Adapting eq. (3), we write the pressure drop across one bubble as

$$\Delta p = \bar{\lambda} \frac{nL_{\mathrm{proj}}}{S} \sigma Ca^{2/3}. \tag{6}$$

All our experimental results are compatible with $\bar{\lambda} \approx 38 \pm 10\%$. This is to be compared with predictions for "1B" structures in square capillaries ($\bar{\lambda} = 1.8$ [7] and $\bar{\lambda} = 6$ [6]). The second value corresponds to a slightly different power law ($\alpha = 0.55$) and is obtained numerically for somewhat larger capillary numbers ($Ca > 3 \cdot 10^{-3}$). A higher experimental value is partly expected due to the role of surfactants. Known to increase flow resistance, these are neglected in most theoretical models (although it has been argued that they would *not* affect the power law exponent [5]). Similarly, although the liquid fraction is not relevant for the scaling behavior, it may give rise to further corrections to the prefactor. In fact it has been pointed out by Xu and Rossen [12] that differences in theoretical models which are related to these effects may change predictions by several orders of magnitude.

Conclusion. – We have performed experiments showing that the dissipation in foam flowing through a narrow rectangular channel is dominated by dissipation associated with the sliding of Plateau borders over the channel walls. This implies that deposition/detachment of films at the walls is the main dissipation mechanism. Our work extends the pressure-velocity scaling (1), which is well known for a train of bubbles through a capillary, to the more complex foam structures considered here. Variations in the prefactor, different from one structure to another, is partially explained in terms of the orientation of Plateau borders with respect to the flow, thus indicating that the contribution of their normal motion is dominant. Given that the aspect ratio of the channel is known to have a very weak effect [7], these results will apply to flow between parallel glass plates. We have therefore, for the first time, provided a detailed characterization of the dissipation processes relevant to quasi-planar rheological experiments. The form of the dissipation law is in good agreement with theoretical predictions for pure water, but the interpretation of the numerical value of the prefactor with respect to such models would require taking into account the effect of the surfactant. These questions, related also to a debate on foam drainage [13], will be addressed in the future by full numerical simulations as well as further experiments using chemically pure products.

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IC thanks *Rennes Métropole* for its financial support. NK wishes to acknowledge a Marie Curie European Fellowship (HPMF-CT-2000-01079) and thanks the GMCM for their hospitality. It is a pleasure to acknowledge discussions with W. ROSSEN, whom we also thank for making his work available prior to publication. We thank E. HENRY and S. MAURUGEON for their participation in preliminary experiments.

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