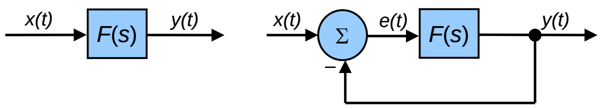
## 9. Regulation and Control

### 9.1 Introduction to feedback

In principle,

- systems designed to produce desired output $y(t)$ for a given input $x(t)$
- such an *open-loop system* should yield the desired output (a):




(a) open-loop system (b) closed-loop (feedback) system

In practice, however, system characteristics change with time

- result of changes in components or environment

Such variations cause changes in the output for the same input:

- highly undesirable in precision systems

Possible solution is to add a signal component to input that will counteract the effects of changing system characteristics / environment

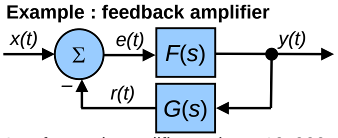
- may be possible to counteract the variations by feeding the output (or some function of the output) back to the input (see (b).

### 9.1 Introduction to feedback (contd)

Feedback systems can be used very widely to minimise unwanted disturbances for example

- random-noise signals in electronic systems
- a gust of wind affecting a tracking antenna
- a meteorite hitting a spacecraft
- a change in slope of a road affecting a car on cruise control

**Example : feedback amplifier**




Effective transfer function :

Let forward amplifier gain = 10 000 ($F(s)$ = $10^4$) and feed back one hundredth of the output to the input ($G(s)$ = $10^{-2}$).

- new gain of amplifier $H = 10^4 / (1 + 10^4.10^{-2}) = 99.01$

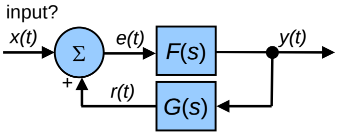
If due to eg change of transistors, forward amplifier gain = 20 000

- gain of amplifier becomes $H = 2\ .10^4 / (1 + 2.10^4.10^{-2}) = 99.5$
- 100% variation in forward gain causes only 0.5% variation in feedback amplifier gain: extremely valuable characteristic

### 9.1 Introduction to feedback (contd)

**Example : feedback amplifier (contd)**

What happens if we add rather than subtract the signal fed back to the input?



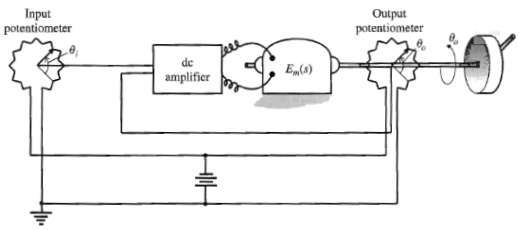

Effective transfer function :

Let forward amplifier gain = 10 000 ($F(s)$ = $10^4$) and feed back $0.9 \times 10^{-4}$ of the output to the input ($G(s) = 0.9 \times 10^{-4}$).

- new gain of amplifier $H = 10^4 / (1 - 0.9 \times 10^{-4} \cdot 10^4) = 10^5$
- If due to eg change of transistors, forward amplifier gain = 11 000
- gain becomes $H = 1.1 \times 10^4 / (1 - 0.9 \times 10^{-4}.\ 1.1 \times 10^4) = 1.1 \times 10^6$
- a 10% increase in forward gain caused a 1000% increase in overall gain: highly undesirable *positive feedback*

### 9.2 Analysis of a simple control system

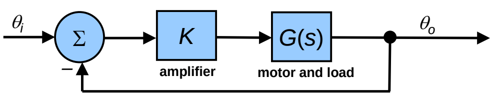
Take as an example an automatic position control system used to control the angular position of a heavy object (eg a tracking antenna)




- input $\theta_i$ is desired angular position of object
- output $\theta_o$ is actual angular position of object measured by potentiometer connected to output shaft
- the difference $(\theta_i - \theta_o)$ is amplified and applied to the motor input
- if $(\theta_i - \theta_o) = 0$ there is no input to the motor and it stops
- if $(\theta_i - \theta_o) \neq 0$ input to motor which will turn the shaft until $(\theta_i - \theta_o) = 0$

### 9.2 Analysis of a simple control system (contd)

Block diagram of control system:




- amplifier gain is $K$ where $K$ is adjustable
- equivalent to (negative) feedback amplifier

**Step input**

- if want to change angular position of object need to apply a step input:

$\theta_i(t) = u(t) \Rightarrow \Theta_i(s) = 1 / s$ and

- let motor (with load) transfer function be $G(s) = 1/(s(s+8))$, yielding

- investigate system behaviour for three values of $K$ : 7, 80 and 16

### 9.2 Analysis of a simple control system (contd)

For $K = 7$

and : system reaches desired angle slowly.

Increase $K$ to 80 in order to speed up system.

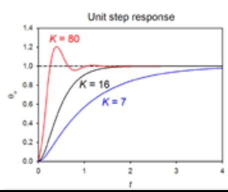
: system reaches position faster but at the expense of oscillations.

To avoid oscillations, characteristic roots of $(s^2 + 8s + K)$ should be real. For $K$>16, roots complex, $K$<16 roots are real: fastest response without oscillations found for $K$ = 16 :

$K$ < 16 – overdamped

$K$ > 16 – underdamped

$K$ = 16 – critically damped



**9.3 Frequency response of an LTI system**

Assume LTI system both causal and stable so all the poles of the transfer function lie in the left half of the s plane.

LTI system response to exponential input is exponential output $e^{st} \Rightarrow H(s)e^{st}$

Setting $s = j\omega$ yields $e^{j\omega t} \Rightarrow H(j\omega)e^{j\omega t}$

$Re(e^{j\omega t})$ = cos $\omega t$ so cos $\omega t \Rightarrow Re[H(j\omega)e^{j\omega t}]$

Can express $H(j\omega)$ in polar form as $H(j\omega) = |H(j\omega)|e^{j\angle H(j\omega)}$

Relationship becomes: $\cos\omega t \Rightarrow |H(j\omega)|\cos[\omega t + \angle H(j\omega)]$

System response $y(t)$ to sinusoidal input cos $\omega t$ is given by

$y(t) = |H(j\omega)|\cos[\omega t + \angle H(j\omega)]$ . System response to cos ($\omega t + \theta$) is

$y(t) = |H(j\omega)|\cos[\omega t + \theta + \angle H(j\omega)]$

Result is only valid for BIBO stable systems – why?

**9.3 Frequency response of an LTI system (cont)**

Summary : for $x(t)$ = cos ($\omega t + \theta$), $y(t) = |H(j\omega)|\cos[\omega t + \theta + \angle H(j\omega)]$

*The amplitude of the output sinusoid is* $|H(j\omega)|$ *times the input amplitude and the phase of the output sinusoid is shifted by* $\angle H(j\omega)$ *with respect to the input phase*.

For instance, for a certain system with $|H(j10)|$ = 3 and $\angle H(j10)$ = –30° amplifies by a factor 3 a sinusoid of frequency $\omega$ = 10 and delays its phase by 30°. E.g. for

$x(t)$ = 5cos (10$t$ + 50°), $y(t)$ = :

$|H(j\omega)|$

- the amplitude gain of the system, also known as *amplitude response* or *magnitude response* of system
- plot of $|H(j\omega)|$ versus $\omega$ shows amplitude gain as a function of frequency

$\angle H(j\omega)$

- the *phase response* of the system
- plot of $\angle H(j\omega)$ versus $\omega$ shows how system modifies the phase of the input sinusoid

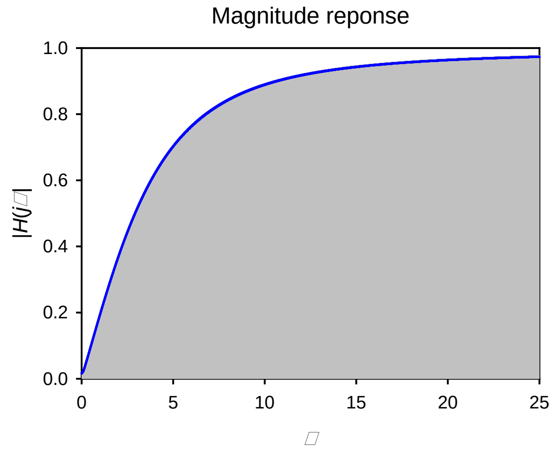
These plots represent the filtering characteristic of the system.

**9.3 Frequency response of an LTI system (cont)**

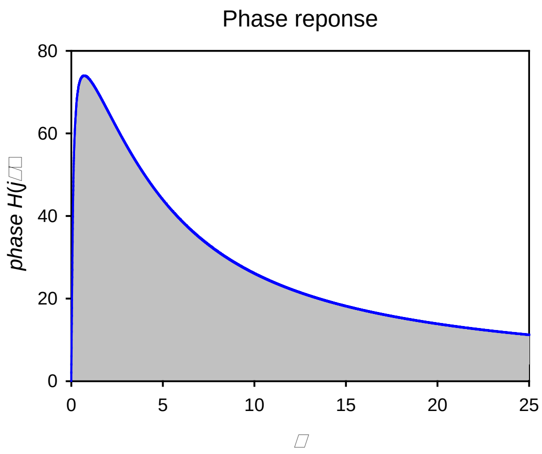
Example: find the frequency response (amplitude and phase response) of a system whose transfer function is $H(s) = \frac{s+0.1}{s+5}$

Find system response $y(t)$ for (a) $x(t)$ = cos 2$t$ (b) $x(t)$ = cos (10$t$ – 50°)

**9.3 Frequency response of an LTI system (cont)**




**9.3 Frequency response of an LTI system (cont)**




**9.3 Frequency response of an LTI system (cont)**

$H(j\omega)$

- contains information on both $|H(j\omega)|$ and $\angle H(j\omega)$
- is called the frequency response of the system
- frequency response plots of $|H(j\omega)|$ and $\angle H(j\omega)$ versus $\omega$ show how system responds to sinusoids of various frequencies

The frequency response plots displayed above show that the system has high pass filtering characteristics.