Signals and systems in physics

0. Introduction

0.1 General Introduction

Signals and systems: study of

- the nature of **signals** – voltages, currents, light intensity etc, and
- how they are affected when they pass through a **system** – that could be a resistor, a capacitor, or some complex amplifier with various band pass filters, or perhaps a light phase modulator.

It is also closely related to the study of **control** theory –

- how to use feedback to modify or control in real time the output signal of a system,
- e.g. temperature in a furnace or blood pressure in body

How is this of interest in physics? Vital for **experimental physics**:

- experimental apparatus gives output signals, but we need to know how these have been transformed – and perhaps how to undo this
- experimental conditions must not only be measured but also precisely controlled
- finally, we must not forget the vital elements of data analysis, parameter fitting and error estimation

0. Introduction

0.1 General Introduction (contd)

Examples from physics: signal processing and recovery

- use of high, low or bandpass filters in both analogue and digital forms to cut noise in experimental signals
- use of phase sensitive detection – lock-in amplifiers – to improve signal-to-noise in spectroscopy
- radiofrequency modulation of laser beams coupled to heterodyne detection to increase sensitivity and permit measurement of very low densities of unstable species

Examples for physics: control of systems

- PID (proportional-integral-derivative) temperature controllers
- quantum coherent control of chemical reactions via phase and amplitude modulation of dispersed ultrafast laser pulses

0. Introduction

0.2 Definition of Signal

Simple definition: a signal is a function of time that carries information.

Fuller definition: A function with.

Arguments:

- time
- place
- time and place

Values: physical quantities such as

- variation of air pressure
- intensity and wavelength of light reflected by an object
- current or voltage...

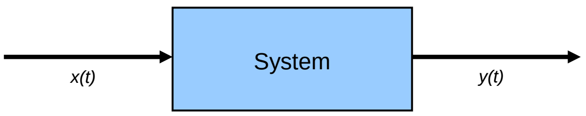
Examples of signals

sound, image, video, electro-cardiogram, flow rate of a river, electromagnetic field

0. Introduction

0.3 Definition of System

An element or elements that transform one or more input signals *x(t)* into one or more output signals *y(t)*:



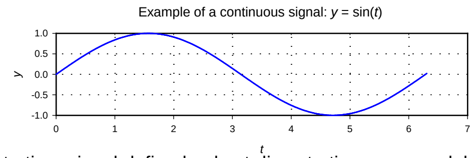
Examples of systems

- amplifier
- microphone, loudspeaker
- radio
- camera
- MP3 player
- translator (human or automatic!)

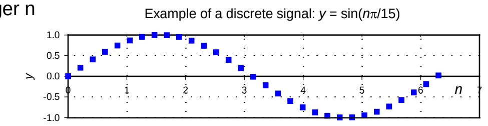
1. Basic signals

1.1 Continuous and discrete signals

Continuous signal just a regular function which can assume any value in some continuous interval (*a*, *b*) e.g. a sine wave



Discrete time signal defined only at discrete times we can label with an integer n

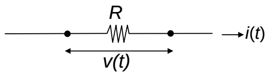


Created from continuous signal by sampling y(t) at regular intervals, T_s

$y[n] = y(nT_s)$

1. Basic signals

1.2 Energy and power in signals



Ohm's law: $v(t) = Ri(t)$

Instantaneous power: $p(t) = v(t)^2/R = Ri(t)^2$

On a per Ohm basis: $p(t) = i(t)^2$

Total energy in Joules: $E = \int_{-\infty}^{\infty} p(t)dt = \int_{-\infty}^{\infty} i(t)^2 dt$

Average power: $P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} i(t)^2 dt$

Energy and power content in arbitrary signal $x(t)$ (may be *complex*): use $|x(t)|^2 = x(t)x^*(t)$ where $x^*(t)$ is complex conjugate

Normalised energy content: $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

Normalised average power: $P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$

or if signal is discrete

$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$ $\quad P = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} |x(n)|^2$

1. Basic signals

1.3 Classification of signals

Energy signal: $0 < E < \infty$, $P = 0$

Power signal: $E = \infty$, $0 < P < \infty$

DC signal: signal with constant value for all times

Periodic signals

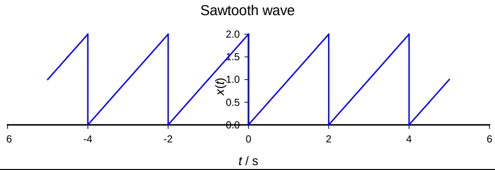
Common. For periodic signal $\exists$ some positive number T_0 (the period): $x(t) = x(t + T_0)$

Fundamental frequency $f_0 = 1 / T_0$ in Hz

For periodic signals, consider energy content over one period, E_O

If $0 < E_0 < \infty$, power signal: $P = E_0 / T_0$

Many types of periodic function possible e.g. sawtooth wave:



by inspection

$T_0 = 2$ s

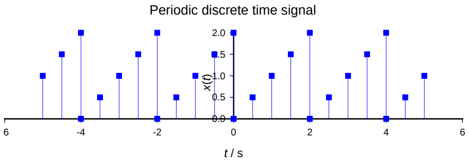
$f_0 = 1 / 2$ Hz

1. Basic signals

1.3 Classification signals (contd)

Periodic discrete time signals also possible:

$x[n] = x[n + N]$: period is N e.g.



Sinusoidal signals

$x(t) = A\cos(\omega t + \theta)$ with A the *amplitude* and θ the *phase angle*.

Fundamental period $T_0 = 2\pi / \omega$, ω the *angular frequency* ($\omega = 2\pi f_0$)

Recall Euler's formula: $e^{\pm j\omega t} = \cos\omega t \pm j\sin\omega t$

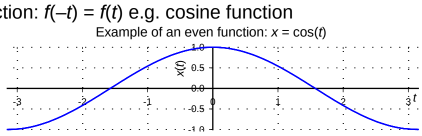
$$\therefore \cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}, \quad \sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

(note use of $j = \sqrt{-1}$ in this subject)

1. Basic signals

1.4 Computing energy

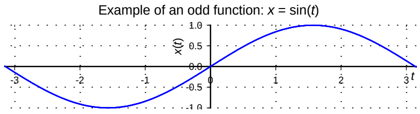
Even function: $f(-t) = f(t)$ e.g. cosine function



For even function $\int_{-a}^{a} f(t)dt = 2\int_{0}^{a} f(t)dt$ and also

$$\int_{-\infty}^{\infty} f(t)dt = 2\int_{0}^{\infty} f(t)dt$$

Odd function: $f(-t) = -f(t)$ e.g. sine function



Generally for an odd function $\int_{-a}^{a} f(t)dt = 0$ or if integral converges $\forall\mathbb{R}$

$\int_{-\infty}^{\infty} f(t)dt = 0$ (note: not the case for $\sin(t)$)

1. Basic signals

1.4 Computing energy (contd)

To summarise: For arbitrary signal $x(t)$

- if $E\left(= \int_{-\infty}^{\infty} |x(t)|^2 dt\right)$ is finite then $x(t)$ is an energy signal
- if $x(t)$ is an energy signal the average power $P = 0$
- a periodic signal is a power signal
- if energy is E_0 over one period T_0 of a periodic signal, then power contained in the signal is $P = E_0 / T_0$
- a signal of finite duration is an energy signal

Example: find the energy content of the following exponentially decreasing signal

$$x(t) = \begin{cases} e^{-2t} & t \ge 0 \\ 0 & t < 0 \end{cases}$$

1. Basic signals

1.5 More on even and odd functions

Even part of a function $x_e(t)$ can be constructed from any function $x(t)$:

$x_e(t) = \frac{x(t) + x(-t)}{2}$ The odd part $x_o(t)$: $x_o(t) = \frac{x(t) - x(-t)}{2}$

Even and odd functions also have the following properties:

- the product of two even functions is even

  $x_1(-t)x_2(-t) = x_1(t)x_2(t)$)
- the product of two odd functions is even

  $x_1(-t)x_2(-t) = [-x_1(t)][-x_2(t)] = x_1(t)x_2(t)$
- the product of an even function times an odd function is odd. For x_1 even and x_2 odd

  $x_1(-t)x_2(-t) = x_1(t)x_2(-t) = -x_1(t)x_2(t)$

Eg: find even and odd components of $x(t) = 2\cos t - \sin t + 3\sin t\cos t$