On the Pricing and Design of Debt-Equity Swaps for Firms in Default

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When they face a significant default event, debt holders can always decide to stay away from bankruptcy[1]. Among available approaches, debt equity swaps consist in swapping the current debt in default for a portfolio made of a new restructured (often reduced) debt and part of the new equity. By doing so, creditors can benefit from at least three advantages. First of all, they avoid immediate liquidation and associated costs. Second, they lower the possible debt overhang. Third, they become equity holders and therefore put a bet on the recovery of the firm with no upside bound. Of course, such a decision will be especially comforted by the knowledge or the conviction that the firm's activity is economically viable. Naturally, the existence of significant growth options in the portfolio of activities may help too.

Empirical studies conclude that swapping debt for equity is a rather popular approach to restructure firms in default. Gilson *and al.* (1990) report, for instance, that equity securities were distributed to creditors in almost 75 percent of the 169 successful restructurings undertaken between 1978 and 1987 they consider, while a similar percentage of restructuring results in a reduction of debt payments. James (1995) finds that 80% of public debt restructurings involve debt-equity swaps against about one in three for the private ones. On his sample covering 1981 through 1990, private lenders make concessions in terms of reducing principal in virtually all the cases[2]. In exchange, banks receive on average one third of the equity when there is no public outstanding, a little more than half otherwise. This result essentially confirms previous findings of Gilson (1990)[3].

Compared to this amount of evidences, the theoretical literature on debt-equity swaps (DES hereafter) appears very thin. Previous research mainly focuses on determinants and motives. None shares our objective to explore design and valuation. This is rather surprising because this subject is of crucial importance for every party involved in private workouts. The main stream views DES as an efficient way to align interests of both parties and to lower agency costs. For Isagawa (2002), DES can limit managerial opportunism too. Sometimes, Debt-Equity Swaps are introduced as an emblematic reorganization scheme implied by out-of-court renegotiation (Fan and Sundaresan (2000) and Ericsson and Renault (2006)). In Fan and Sundaresan (2000), DES are surprisingly modelled as if it was a sharing of the market value of the firm's assets. In agreement with factual observations, one will assume in what follows that only new equity is shared among claimants.

Our paper contributes to the literature by solving the simultaneous problem of design and pricing, claimholders face, when debt holders accept to stay away from liquidation. We assume that creditors control the financial reorganization so that they are incited to optimize their own positions. We demonstrate that there is no straightforward solution in the sense that admissible DES solve simultaneously a system of two equations with three unknown variables. Assuming that creditors do not want to expropriate current equity holders from the firm, we show that an extra condition is needed to find an optimal (non corner) solution. We then investigate DES in different contexts. Among other things, simulations conclude that DES significantly increase the probability of being reimbursed of the remaining face value in the next future.

The rest of the paper is organized as follows. In Section I, we develop our structural framework and introduce DES. Section II introduces and prices ad hoc as well as admissible DES. Section III studies the existence of admissible DES, whereas both designs are contrasted in Section IV. Section V questions how to design an optimal DES. -Section VI offers a numerical illustration.

I. The framework

Our analysis adopts the continuous time framework of Black, Scholes and Merton (1973, 1974), except that there are bankruptcy costs. Financial markets are therefore almost perfect, complete and trading takes

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place continuously. There exists a riskless asset paying a known and constant interest rate denoted by r. There are neither taxes, nor transaction costs. The considered firm is financed by equity and a single debt whose debt maturity and face value are denoted by T_1 and F_1. Debt takes the form of a zero coupon bond. The firm's assets value is assumed to be correctly described, under the risk neutral measure, by:

$$dV = rVdt + \sigma_1 VdW \tag{1}$$

where W is a Brownian motion. σ_1 denotes the firm's assets volatility and stands for the level of business risk[4]. At time T_1, equity holders receive $\max(V_{T_1} - F_1; 0)$ *i.e.* the pay-off of a call option written on the firm's assets value. For their part, debt holders receive the promised face value if $V_{T_1} \geq F_1$. Otherwise, they can obtain the value of the liquidated firm's assets denoted by βV_{T_1} if $\beta \in [0,1[$ is the realization rate. $(1-\beta) \in]0,1]$represents bankruptcy costs. In view of empirical results exposed in Bris *and al.* (2006), these costs can be very substantial. It must be understood that, to account for these costs, the firm is worth less than the sole firm's assets value. If liquidation costs were null ($\beta = 1$), the firm value would be strictly equal to the firm's assets value. The value of the restructuring is in our setting the present value of liquidation costs to be saved. If liquidation costs were null ($\beta = 1$), there would be no incentive in our setting to postpone decision to bankrupt.

Let's now assume that debt holders face a default $(V_{T_1} < F_1)$ and that they intend to enter a debt equity swap. They now face the problem to choose appropriately a) the amount A of face value exchanged for equity, b) the proportion θ of equity they receive in exchange of A and c) the set Θ of parameters that describes the financial set up. In our setting, Θ contains τ the time-to-maturity of the new rescheduled zero-coupon bond and $F_1 - A$ the due face value to be paid at τ. Because creditors become equity holders, τ can be interpreted as the length of their investment *period*. Given that the due face value F_1 is known at time T_1, debt equity swaps are fully characterized by three structural parameters: A, θ and τ. Note that hereafter, we will interchangeably use τ or $T_2 = T_1 + \tau$, the investment *horizon*.

Two structural parameters deserve additional comments with regards values claim holders should consider: these are the investment period and the proportion of equity received. One expects T_2 to be a moderate investment horizon. A too short period can prevent the firm to recover, while a too long period is not desirable either because there is no interim payment in our setting. Creditors will therefore bound T_2 in order to be effectively reimbursed. This implies that they can remain equity holders for years. As a matter of facts (see James (1995)), banks are used to maintain a substantial stake years after the restructurings even if various legal and regulatory factors can constraint them from stocks holdings. The participation θ necessary lies between 0% and 100% ($\theta \in [0,1]$), but some specific levels are more meaningful than other in terms of voting rights. $\theta = 34\%$, *e.g.*, allows debt holders to limit current equity holders decision whereas $\theta = 49\%$ maintains the leadership of current equity holders on the firm's management decision. When θ is set to zero (while A being strictly positive), debt holders simply forgive part of their debt. When θ is set to one, debt holders expropriate current equity holders. In what follows, one assumes that the aim of debt holders is not to evict current equity holders so that the value of θ remains lower than fifty percent. An economic rationale for this may be that current equity holders/managers have special skills and/or unique resources concerning the business so that it could be costly to make them leave the firm.

By setting A, θ and τ appropriately, creditors can design the debt-equity swap so as to maximize their wealth. For their part, current equity holders are always better off with rescheduling. They indeed receive a new claim whose financial value is strictly positive, whereas current equity of the firm in default is worth nothing. In addition, they can maintain a significant degree of control on the firm's management (evaluated by $1 - \theta$). The bottom line is that both claimholders have strong incentives to enter a cooperative restructuring.

II. The pricing of debt equity swaps

For debt holders, swapping debt in default for equity means substituting the known payoff of βV_{T_1} for a new portfolio made of debt and part of the equity. Using notations introduced in section 1, the net gain function for creditors, denoted H, may be written:

$$\begin{aligned} &H(V_{T_1}, F_1, T_2; A, \theta) = D_n(V_{T_1}, F_1 - A, T_2 - T_1) \\ &+ \theta C_n(V_{T_1}, F_1 - A, T_2 - T_1) - \beta V_{T_1} \end{aligned} \tag{2}$$

where

$$\begin{aligned} &D_n(V_{T_1}, F_1 - A, T_2 - T_1) = \\ &e^{-r(T_2 - T_1)} E^Q_{T_1}\left[\beta V_{T_2} 1_{V_{T_2} < F_1 - A} + (F_1 - A) 1_{V_{T_2} \geq F_1 - A}\right] \end{aligned} \tag{3}$$

stands for the new debt with a lower face value and

$$\begin{aligned} &\theta C_n(V_{T_1}, F_1 - A, T_2 - T_1) = \\ &\theta e^{-r(T_2 - T_1)} E^Q_{T_1}\left[\left(V_{T_2} - (F_1 - A)\right)^+\right] \end{aligned} \tag{4}$$

is the value of new equity received in exchange of A. Standard computations under the risk neutral measure Q then yield to:

$$\begin{aligned} &H(V_{T_1}, F_1, T_2; A, \theta) = \\ &-\beta V_{T_1} N\left[d_{1, V_{T_1}/(F_1 - A)}(T_2 - T_1)\right] \\ &+ (F_1 - A) e^{-r(T_2 - T_1)} N\left[d_{2, V_{T_1}/(F_1 - A)}(T_2 - T_1)\right] \\ &+ \theta\Big(V_{T_1} N\left[d_{1, V_{T_1}/(F_1 - A)}(T_2 - T_1)\right] \\ &- (F_1 - A) e^{-r(T_2 - T_1)} N\left[d_{2, V_{T_1}/(F_1 - A)}(T_2 - T_1)\right]\Big) \end{aligned}$$

where $d_{1,x}(t) = \dfrac{\ln x + \left(r + \frac{1}{2}\sigma_1^2\right)t}{\sigma_1\sqrt{t}}$, $d_{2,x}(t) = d_{1,x}(t) - \sigma_1\sqrt{t}$ and N is the standard cumulative

distribution function. This last equation may be rewritten in a more compact form:

$$H(V_{T_1}, F_1, T_2; A, \theta) = (\theta - \beta) V_{T_1} N\left[d_{1, V_{T_1}/(F_1 - A)}(T_2 - T_1)\right] + (1-\theta)(F_1 - A)e^{-r(T_2 - T_1)} N\left[d_{2, V_{T_1}/(F_1 - A)}(T_2 - T_1)\right]. \quad (5)$$

This expression highlights that a DES restructuring is a bet on a future recovery of the firm.

It is easy to show that the net present value of the restructuring is in our setting: $(1-\beta) V_{T_1} N\left[d_{1, V_{T_1}/(F_1 - A)}(T_2 - T_1)\right]$. This is the present value of the liquidation costs and also the maximum net gain debt holders can receive if they decide to expropriate former equity holders ($\theta = 1$). Clearly, there is no reason to postpone liquidation if: $\beta = 1$.

As a special case, one may verify first that when the investment period is infinite, the net gain function is worth $(\theta - \beta) V_{T_1}$ implying that, for very long term positions, creditors are incited to swap debt for equity when θ is greater than β.

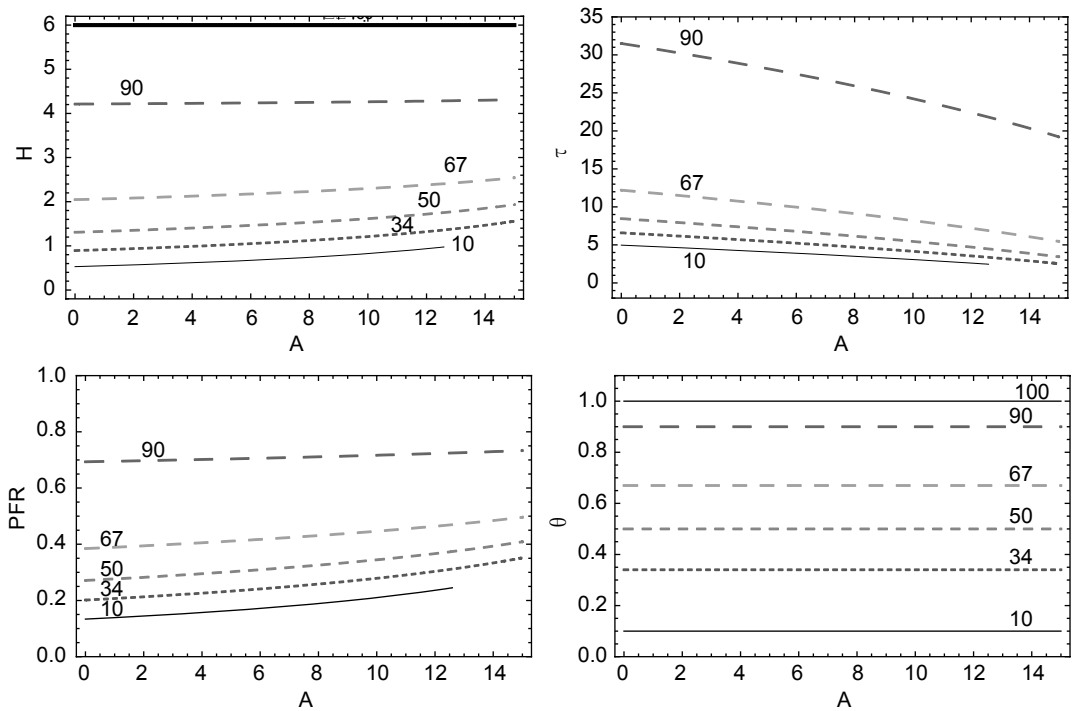
Because debt holders control the restructuring, the new financial set up is designed by setting structural parameters so as to maximize the above net gain function. A detailed exploration of H, viewed as a function of T_2, reveals that, for most arbitrary values of A and θ, it admits a maximum with positive value (see the appendix for a detailed discussion of this point). Given A and θ, debt holders should therefore be able to determine the length of the investment period in most of cases by solving[5]:

$$\tau(A, \theta) = \arg \max_{T_2 \in [T_1, +\infty[} H(V_{T_1}, F_1, T_2; A, \theta) - T_1. \quad (6)$$

This length of period may be viewed as the effort consented by debt holders. Note finally that solving $\max_{\substack{T_2 \in [T_1, +\infty[\\ \theta \in [0,1] \\ A \in [0, F_1[}} H(V_{T_1}, F_1, T_2; A, \theta)$ is trivial. Indeed, creditors will first expropriate current stockholders by setting $\theta = 1$ and then immediately give up their debt so as to sell the firm in one piece to new investors. At the bottom line, they save $H(V_{T_1}, F_1, T_1; A, 1) = (1-\beta) V_{T_1}$, liquidation costs.

To illustrate what the above procedure implies, it is worth to run simulations in a base case environment. Let it be a firm whose assets value is $V_{T_1} = 20$, whose business risk is $\sigma = 20\%$. Let's assume that the debt face value is $F_1 = 40$ so that the firm is currently in a severe default situation. Liquidation may be decided with a realization rate equal to $\beta = 70\%$. The level of the interest rate is currently assumed to be $r = 6\%$. Figure 1 plots, as functions of A and for different and independent θ, the net gain function $H(T(A, \theta); A, \theta)$, the investment period $\tau(A, \theta)$ and the probability of being fully repaid $PFR(A, \theta) = Q\left[V_{T(A,\theta)} \geq F_1 - A \,\middle|\, V_{T_1}\right] = N\left[d_{2, V_{T_1}/(F_1 - A)}(T(A, \theta) - T_1)\right]$. The lower right graph conveys that the received portion of new equity θ is chosen arbitrarily and independently from A. We choose meaningful values for θ (i.e. 10%, 34%, 50%, 67%, 90% and 100%) that are represented by thin lines.

Figure 1. Ad hoc debt-equity swaps



These graphs plot as functions of A: the net gain function $H(T(A, \theta); A, \theta)$, the investment period $\tau(A, \theta)$ and the probability of being fully repaid as functions of the forgiven face value $PFR(A, \theta)$ when the received portion of new equity θ is chosen arbitrarily and independently from A (see the lower right graph). Thin and dashed lines are associated to fixed portions θ (set to 10%, 34%, 50%, 67%, 90% and 100%). Other parameters are: $V_{T_1} = 20$, $F_1 = 40$, $\sigma = 20\%$, $r = 6\%$, $\beta = 70\%$.

Left graphs suggest first of all that, for a given portion θ, the net gain function and the probability of being fully repaid increases only moderately with A. But the right upper graph reveals that the corresponding investment period may decrease significantly. Because the benefit from this restructuring is almost insensitive to the exchanged value A, creditors may conclude that designing a DES is merely a question of choosing the length of the investment period.

Designing debt-equity swaps with the above procedure is however largely naïve or *ad hoc* because A and θ should not be chosen independently. It is clear that debt holders should not accept to give up part of their debt without an eye on the portion of the equity they receive (and on the value this represents). The relation between the amount A, the portion θ and the investment period τ is therefore far more subtle than assumed earlier. Ideally, structural parameters should be determined simultaneously.

A debt-equity swap is *admissible* for creditors if i) it ensures that the received portion of equity covers the forgiven amount of face value, and ii) if the associated net gain function is maximum. Hence, a debt-equity swap is admissible if and only if:

$$A = \theta\, \mathbf{C}_n\big(V_{T_1}, F_1 - A, T(A,\theta) - T_1\big). \tag{7}$$

where the optimal extension period $T(A,\theta)$ solves the equation:

$$T(A,\theta) = \arg\max_T H\big(V_{T_1}, F_1, T\,;A,\theta\big) \tag{8}$$

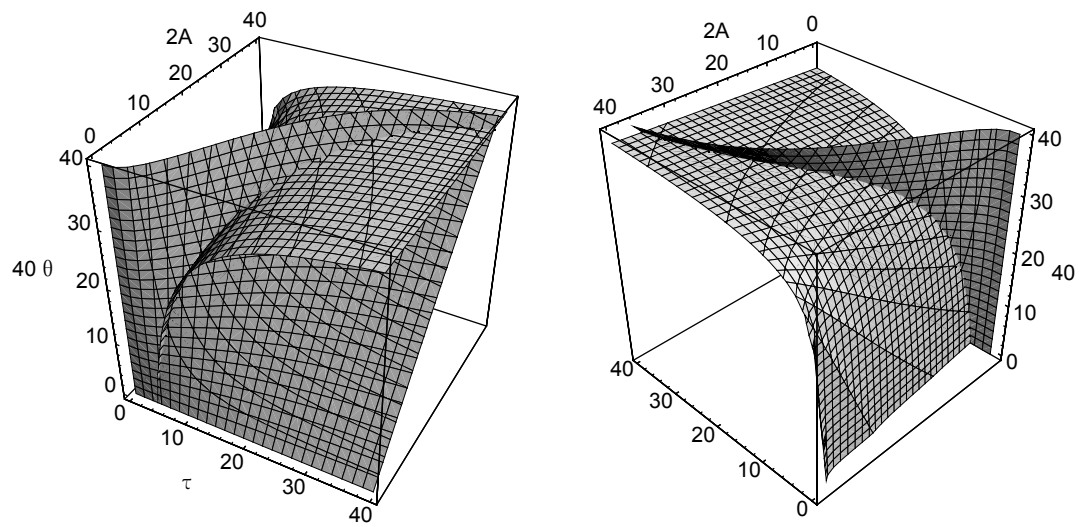
The equation (7) ensures that if creditors sell their part of equity, they receive immediately an amount equal to A which is the amount of face value they exchange. As a result, the only effective effort creditors consent is to delay the repayment of the distressed debt. We can add that the new debt price is given by $H\big(V_{T_1}, F_1, T(A,\theta)\,;A,\theta\big) - A + \beta V_{T_1}$. For their part, equity holders also benefit from the debt equity swap because they receive a new claim whose value is strictly positive. Resembling to a call option, this claim is worth $(1-\theta)\mathbf{C}\big(V_{T_1}, F_1 - A, \tau(A,\theta)\big)$.

III. Existence of admissible debt equity swaps

Existence of admissible debt equity swaps is ensured to the extent there exist structural parameters $\{A,\tau,\theta\}$ that solve the equations (7) and (8) simultaneously. To investigate whether admissible debt-equity swaps exist, we proceed as follows. We consider two functions of structural parameters, these are $\theta\mathbf{C}\big(V_{T_1}, F_1 - A, \tau\big) - A$ and $\dfrac{\partial H\big(V_{T_1}, F_1, T\,;A,\theta\big)}{\partial T}$, and then we produce three-dimensional plots of their respective zero values. Figure 2 puts together a couple of standpoints. Looking in the axis of τ, the "wall" stands for the triplets $\{A,\tau,\theta\}$ solving $\theta\mathbf{C}\big(V_{T_1}, F_1 - A, \tau\big) - A = 0$ whereas the "fall"corresponds to triplets associated to $\left.\dfrac{\partial H\big(V_{T_1}, F_1, T\,;A,\theta\big)}{\partial T}\right|_T = 0$. The intersection points out the set of admissible structural parameters, or more properly structural parameters of *admissible* debt equity swaps.

Figure 2 uses base case parameters, θ and A ranging respectively from 0 to 1 and from 0 to 20. Note that, for visualization purposes, we scale up θ by a factor 40 and A

Figure 2. The set of admissible debt-equity swaps



These graphs show, under a couple of viewpoints, the triplets $\{A,\tau,\theta\}$ solving $\theta\mathbf{C}(V_{T_1}, F_1 - A, T - T_1) - A = 0$ (the "wall" in the τ -axis) and those solving $\left.\dfrac{\partial H(V_{T_1}, F_1, T\,;A,\theta)}{\partial T}\right|_T = 0$ where $T = T_1 + \tau$ (the "fall" in the τ -axis). The intersection corresponds to the structural parameters of admissible debt equity swaps. θ ranges from 0 to 1 and A from 0 to 20 but, for visualization purposes, we multiply θ by 40 and A by 2. Other parameters are: $V_{T_1} = 20$, $F_1 = 40$, $\sigma = 20\%$, $r = 6\%$, $\beta = 70\%$.

by a factor 2. The intersection of the two surfaces proves that there exist some triplets $\{A, \tau, \theta\}$ that solve equations (7) and (8) simultaneously. Actually, the knowledge of one parameter implies some values for the two other ones. For instance, given that $A = 5$, we get $\tau = 12.36$ and $\theta = 72.84$ to ensure that the DES is admissible. Given that $\tau = 15$, we get $A \approx 7.20$ and $\theta \approx 80.1$ and so on. These graphs in addition reveal that some admissible debt equity swaps require very special and extreme values for structural parameters, *e.g.*, a very long (and unrealistic) investment period associated to a complete eviction of current equity holders.

IV. Admissible design *versus* ad hoc designs: some simulations

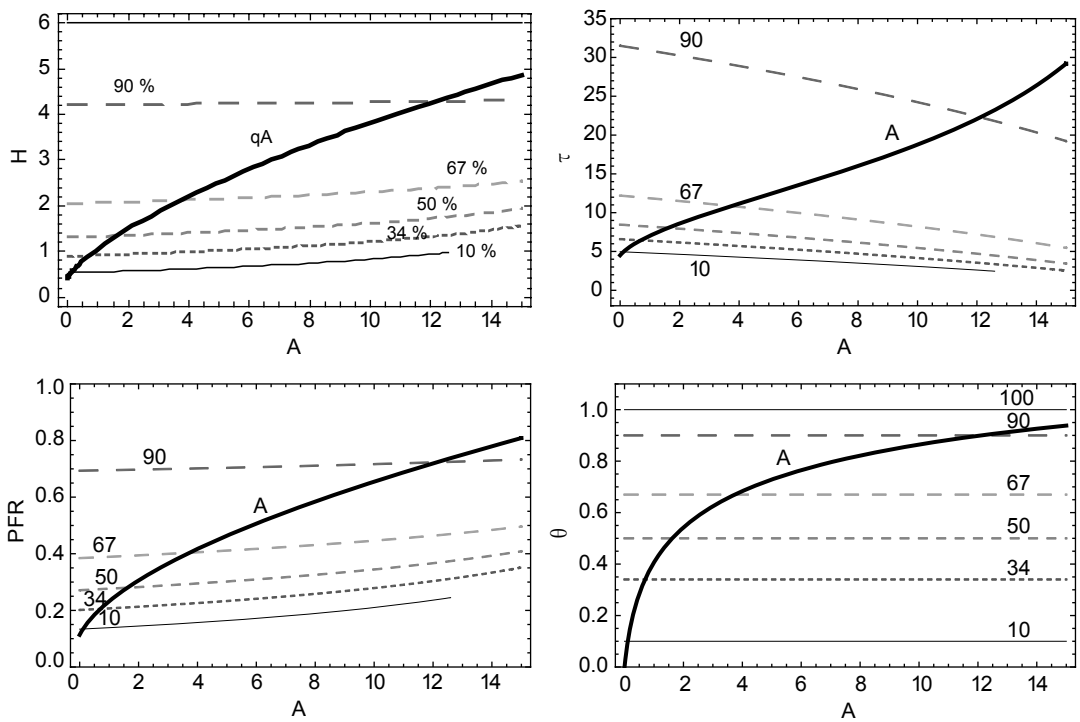
In this section, we analyze admissible DES by comparison to *ad hoc* designs. To this end, Figure 3 reconsiders Figure 1 and adds, in bold lines, values corresponding to the portion $\theta(A)$ that solves equations (7) and (8) simultaneously.

Figure 3 first illustrates how different is our method to design debt equity swaps from an *ad hoc* methodology. It can be observed that *ad hoc* designs are admissible in very few cases. These cases are situated where thin lines intersect bold lines. For a portion equal to $\theta = 90\%$, net gain functions of *ad hoc* designs essentially overestimate those of admissible DES *i.e.* $H(T(A, 90\%); A, 90\%) > H(T(A, \theta(A)); A, \theta(A))$ in most cases. As it is demonstrated by left graphs, reasons for this are twofold. The first reason is that debt holders grant a larger delay $\tau(A, 90\%) >> \tau(A, \theta(A))$ (see the left upper graph). The second reason is that they obtain a greater and undue portion of equity $90\% >> \theta(A)$ in most of cases (see the left lower graph).

Figure 3 also indicates that, by increasing the amount A, admissible DES have strong ability (and stronger than *ad hoc* designs) to increase the net gain function (upper left graph) and the probability of being fully repaid of the remaining face value (lower left graph) to the extent creditors accept to make a significant effort to wait (upper right graph) and are able to capture a significant portion of equity (lower right graph).

Left graphs of Figure 3 may suggest creditors that they are better off by exchanging a large amount of the face value, but this conclusion is amended by bold lines on right graphs (that are associated to admissible DES). Associated investment periods may be very long (more than 25 years) making such a choice less realistic. This comes from the equity component of the DES.

Figure 3. Designing admissible debt-equity swaps *versus ad hoc* solutions



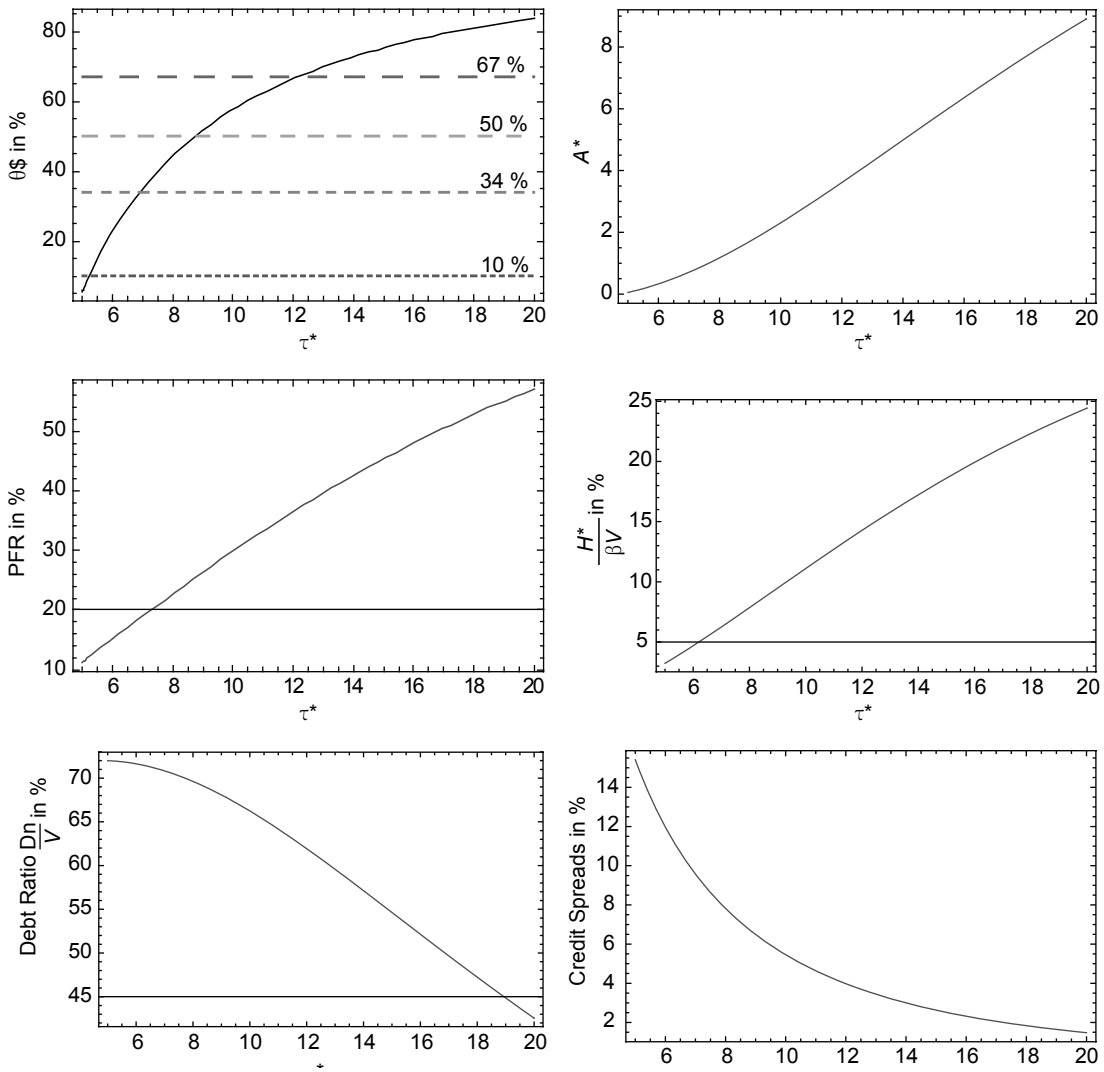
These graphs plot as functions of A: the net gain function $H(T(A, \theta); A, \theta)$,, the investment period $\tau(A, \theta)$ and the probability of being fully repaid as functions of the forgiven face value $PFR(A, \theta)$ when the received portion of the new equity is arbitrarily or optimally chosen. The fourth graph reminds that the portion of new equity received by the debt holders is either arbitrarily constant or equal to $\theta(A)$. Thin and dashed lines are associated to fixed portions θ (set to 10%, 34%, 50%, 67%, 90%, even 100%). Bold lines draw $H(T(A, \theta(A)); A, \theta(A))$, $\tau(A, \theta(A))$, $PFR(T(A, \theta(A)), A, \theta(A))$, and $\theta(A)$ respectively. Other parameters are: $V_{T_1} = 20$, $F_1 = 40$, $\sigma = 20\%$, $r = 6\%$, $\beta = 70\%$.

Interestingly, the lower right graph shows that $\theta(A)$ is a non-linear increasing function of A and secondly that meaningful thresholds of participation in equity (10%, 34%, 50% and 67%) are reached for small values of A[6]. It therefore suggests that former equity holders can be evicted even in soft restructuring scenarii. Our model predictions are broadly consistent with factual observations. First, it shows that former equity holders can be expropriated, even for moderate restructuring as in the Marconi's case. Second, inspection of this graph reveals that, in our base case environment, exchanging 10% of the face value (exactly 4 out of 40) implies receiving 67% of the new equity. Strikingly this same magnitude has been observed in the famous restructuring of Eurotunnel in 1997. As a matter of facts, 1 Billion euros out of 9.1 Billion euros were exchanged for about 60% of the new equity capital.

V. How to design an optimal debt equity swap?

The key question is now to answer whether some optimal debt equity swap may be favoured among the set of admissible ones. It has already been observed with equations (7) and (8) and Figure 2 that the knowledge of one parameter implies the values of the two other parameters. In addition, the Figure 3 indicates that no specific value for the amount A is numerically more desirable than others, when one assumes that debt holders

Figure 4. Designing Debt Equity Swaps with respect to the investment period and the new financial set up



These four graphs plot as a function of the investment period τ^*: the portion θ^*, the exchanged amount A^*, the probability of being fully repaid of the remaining face value PFR^*, the relative net profit, the debt ratio and the credit spreads implied by a debt equity swap that solves equations (7), (8) and (9). Other parameters are: $V_{T_1} = 20$, $F_1 = 40$, $\sigma = 20\%$, $r = 6\%$, $\beta = 70\%$.

do not want to take over the firm, nor they want to wait for an infinite time to be reimbursed. Hence, an extra and exogenous condition must be added. This section discusses and explores possible constraints affecting one structural parameter: A^*, τ^* or θ^*. We rank them by order of computational complexity.

In view of Figure 3, the simplest approach for debt holders is to choose arbitrarily a value for the amount $A = A^*$. Other structural parameters are then simply computed by $\theta^* = \theta(A^*)$ and $\tau^* = \tau(A^*, \theta(A^*))$ and the associated net gain function becomes $H^* = H\big(V_{T_1}, F_1, T(A^*, \theta(A^*)); A^*, \theta(A^*)\big)$. It is sufficient here to examine different graphs of Figure 3 to obtain proper values.

There exists however a second (and perhaps more realistic) approach that consists in selecting first a specific participation θ^*. Theoretically, θ^* can be set to all values in the interval $[0,1]$ but some values, such as 10%, 34%, 50% and 67%, are more likely to be chosen than others (incidentally, they have been retained in graphs of Figure 3). To design the associated optimal financial set up, debt holders can imply the corresponding amount A^* graphically from the lower right graph[7]. There is no ambiguity on the graph because $\theta(A)$ is a strictly increasing function of A with an image in $[0,1]$ (meaning that $\theta(A)$ is a one-to-one function of A on its domain of definition). Hence, the solution is unique and may be denoted by $A^* = \theta^{-1}(\theta^*)$. The third structural parameter is then obtained by finding $\tau(\theta^{-1}(\theta^*), \theta^*)$ on the bold line of the upper right graph. The resulting net gain function for debt holders, on the bold line of the upper left graph, is $H^* = H\big(V_{T_1}, F_1, T(\theta^{-1}(\theta^*), \theta^*); \theta^{-1}(\theta^*), \theta^*\big)$ (with $T = (\theta^{-1}(\theta^*), \theta^*) = T_1 + \tau(\theta^{-1}(\theta^*), \theta^*)$).

Finally, debt holders may rationally consider a maximum investment period τ^* with a focus on the capital structure in mind. The extra time-to-maturity is then expressed as

$$\tau(A, \theta) = \tau^*. \tag{9}$$

Despite its apparent simple form, caution must be advised in interpreting the above condition. It actually nests a couple of ideas. First of all, contrasting with the two first approaches, the investment period is no more instrumental here but rather a key dimension of the design. Second, debt-equity swaps are designed (by choosing A and θ accordingly) so as to make this horizon period optimal in the sense of equation (6) $\Big(\tau(A, \theta) = \arg \max_{T_2 \in [T_1, +\infty[} H\big(V_{T_1}, F_1, T_2; A, \theta\big) - T_1\Big)$. Given an investment period of τ^*, two approaches are possible to determine associated structural parameters. Graphically, we can imply other two structural parameters from Figure 3. The amount A^* can be deduced from the upper right graph and the portion of equity from the lower right graph. More formally, one finds $A^* = \tau^{-1}(\tau^*)$ and $\theta^* = \theta(A^*) = \theta(\tau^{-1}(\tau^*))$. The corresponding net gain of this debt equity swap is then $H^* = H\big(V_{T_1}, F_1, T^*; \tau^{-1}(\tau^*), \theta(\tau^{-1}(\tau^*))\big)$ where $T^* = T_1 + \tau^*$ and the probability of being fully repaid of the remaining face value is $PFR^* = PFR(T^*, A^*, \theta^*)$.

An alternative approach is to directly solve the system formed by the equation (7), (8) and (9). To illustrate the feasibility of this approach, Figure 4 explores several variables related to the newly (and optimally) designed capital structure as functions of the investment period τ^*.

The upper graphs of Figure 4 inform on other structural parameters: the participation θ and the exchanged amount A. The middle graphs Figure 4 highlight how worthwhile is our solution by plotting the associated probability of being fully repaid of the remaining face value and the relative net profit. Debt-equity swaps imply different capital structures to analyze so lower graphs describe corresponding financial sets up through the debt ratio and the credit spreads.

VI. A numerical illustration

This section explores numerically various new financial sets up and DES given that debt holders favour the θ – based criterion. θ^* is set to 50% so that the new equity is halved and shared among parties. $A^* = \theta^* \times Eq^*$ is therefore the value received by both debt holders and former equity holders. A^* is also the total net gain of former equity holders because the equity of a firm in default is worth nothing. The total value of equity is $Eq^* = 2A^*$.

Table 1 considers fifty different default contexts (among which that exposed in precedent figures). It provides structural parameters of the optimal DES (*i.e.* $A^* = \theta^{-1}(\theta^*), \tau^*(\theta^{-1}(\theta^*), \theta^*)$), the net gain $H^* = H\big(V_{T_1}, F_1, T(\theta^{-1}(\theta^*), \theta^*); \theta^{-1}(\theta^*), \theta^*\big)$ as well as the probability that the remaining face value will be fully repaid in the next future $PFR^* = PFR(T(\theta^{-1}(\theta^*), \theta^*), \theta^{-1}(\theta^*), \theta^*)$. The length of the investment period τ^*, purely instrumental here, is (as usual) interpreted as the effort consented by debt holders.

Let's first examine a specific situation where, say, $\beta = 60\%$ and $V_{T_1} = 30$. Simulations show that the net gain of the DES is $H^* = 4.746$ proving that debt holders are better off to swap debt for equity. The face value of the new debt is equal to: $F^* = F_1 - A^* = 40 - 2.94 = 37.06$. The effort remains moderate because debt holders grant a delay of $\tau^* = 4.57$ years only. Interestingly, the probability of being fully repaid of the remaining face value F^* is $PFR^* = 47.34\% \approx \frac{1}{2}$. This probability is significant compared to the fact that debt holders currently face a default and that they can incur a loss if they decide liquidation immediately. Beyond this specific context, Table 1 shows that H^* is systematically positive, meaning that it is worth for creditors to swap debt for equity in all situations. In every case, concessions (τ^*) appear rather weak and probabilities of recovery indicate a significant departure from default. Nevertheless, it can be pointed out that the firm is still maintained in financial distress. These evidences prove that debt-equity swap is a rather powerful solution in the hand of creditors even if some further device could help fostering the firm's recovery.

Table 1. Pricing optimal debt equity swaps that ensure a target participation of $\theta^* = 50\%$

Realization Rate β		Firm Value at Time of First Default									
		20	22	24	26	28	30	32	34	36	38
80	A^*	0.81	0.91	1.01	1.10	1.18	1.25	1.29	1.29	1.21	0.96
	τ^*	5.72	4.98	4.29	3.65	3.05	2.48	1.94	1.42	0.91	0.41
	H^*	0.498	0.614	0.749	0.907	1.094	1.318	1.588	1.926	2.365	2.987
	PFR^*	0.1765	0.1997	0.2244	0.2508	0.2791	0.3097	0.3430	0.3798	0.4210	0.4674
70	A^*	1.62	1.76	1.89	1.99	2.07	2.12	2.12	2.04	1.84	1.36
	τ^*	8.04	7.02	6.07	5.19	4.35	3.56	2.80	2.06	1.34	0.61
	H^*	1.343	1.584	1.851	2.149	2.486	2.868	3.309	3.828	4.459	5.262
	PFR^*	0.2802	0.3038	0.3280	0.3527	0.3783	0.4049	0.4326	0.4616	0.4916	0.5197
60	A^*	2.41	2.58	2.73	2.84	2.92	2.94	2.90	2.76	2.44	1.76
	τ^*	10.14	8.88	7.70	6.60	5.56	4.57	3.61	2.68	1.75	0.80
	H^*	2.477	2.856	3.267	3.714	4.204	4.746	5.351	6.039	6.837	7.770
	PFR^*	0.3618	0.3838	0.4059	0.4282	0.4506	0.4734	0.4964	0.5195	0.5416	0.5571
50	A^*	3.14	3.33	3.50	3.62	3.69	3.70	3.63	3.42	3.01	2.14
	τ^*	12.07	10.59	9.21	7.92	6.70	5.52	4.39	3.28	2.17	1.01
	H^*	3.811	4.338	4.900	5.502	6.149	6.850	7.618	8.469	9.421	10.455
	PFR^*	0.4265	0.4467	0.4667	0.4865	0.5062	0.5258	0.5452	0.5639	0.5803	0.5868

This table presents the amount of face value optimally exchanged by debt holders for equity (on the first row), the associated optimal extension period (on the second row), their net gains from optimally swapping debt for equity (on the third row) and the probability of being fully repaid (on the fourth row), for different values of the realization rate (β in percentage) and values of the firm assets at the time of default V_{T_1}. Remind that A^* is also the value received by former equity holders. Other parameters are: $F_i = 40$, $\sigma_1 = 20\%$, $r = 6\%$.

VII. Conclusion

This paper analyses debt equity swaps with a special focus on their pricing and design. Such financial reorganization tools are useful when creditors want to avoid straight liquidation of the firm in default. We highlight that three parameters (only) can characterize salient features of this restructuring solution. These are the amount of exchanged debt, the participation level in the new equity and the duration of the investment period. We assume that debt holders control the financial restructuring with no intent to take over the firm or expropriate current equity holders. We then solve the debt holders' design problem. We find that there exists a set of admissible solutions and we highlight different ways to design debt equity swaps. We explore optimal restructuring solutions when creditors consider an additional request on the exchanged amount of debt, the investment horizon or the participation level in the new equity. Simulations conclude that the debt equity swap is an interesting method to restructure distressed debt. Computations we run may provide useful quantitative benchmarks to understand and design real-life restructurings.

Thanks

Part of this paper was presented at the AFFI Paris International conference organized in December 2007 and at the EFMA 2006 conference in Madrid. We thank participants and Areski Cousin for comments and discussion. This paper also benefits from comments of Jean-Paul Décamps and Patrice Fontaine. In addition, the first author also thanks Elizabeth Whalley and Marco Realdon for stimulating exchanges. ■

1 There are many ways to financially reorganize distressed firms. Among others, one finds, the debt forgiveness, the reduction of interest or principal and the extension of maturity. For more details on these, one may consult Asquith *and al.* (1994), Gilson *and al.* (1990), Longstaff (1990), John (1990) and the references herein. *In Banque et Marchés*, Moraux and Navatte (2006) have studied straight debt rescheduling.

2 On average, the reduction of principal represents about 40% of the loan amount. We claim in what follows that a loan reduction should not be chosen arbitrarily but should rather be closely related to the other parameters characterizing debt-equity swaps.

3 In some extreme cases, equity holders are simply expropriated. In this respect, the restructuring of Marconi in 2002 (where 99.5% of the new equity went to creditors) is typical. The Independent wrote, the 29-th of august 2002: "In a corporate collapse of this type, bankers and bondholders hold all the cards. They don't have to give anything to shareholders at all. [...] That there is a salvageable business in there somewhere is evidenced by the fact that bankers have gone the debt for equity route, rather than putting the company into liquidation". One will not insist on such extreme cases, but they can be understood within the following analytical setting.

4 Such a dynamic may be viewed as inadequate for the considered context. Moraux (2004), *e.g.*, offers arguments for adding jumps and a stochastic volatility. This assumption remains however highly standard in the literature and constitutes a benchmark impossible to circumvent.

5 Remind that this ad hoc way is not what we recommend in the present paper. We favour the admissible design that we will describe below.

6 Note that this graph also demonstrates that the marginal interest of exchanging one more euro to evict former equity holders declines.

7 An alternative approach is to directly solve the system formed by the equations (7), (8) with an additional equation: $\theta = \theta^*$.

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Appendix

This appendix discusses the net gain function described by equations (2) and (5) in some details. First of all, it must be pointed out that this function is continuous with respect to the maturity T_2 and that it admits a soft behaviour. Second, although simulations show that its convexity may change for some range of parameters, its graph is merely humped shape. Third, the net gain function allows however no simple formula for its derivatives with respect to T_2, and no closed form solution for τ.

The function described by equations (2) and (5). may be viewed as an extension of that of Longstaff (1990). Setting θ and A to zero provides the Longstaff's function:

$$H(V_{T_1}, F, T_2; A = 0, \theta = 0) = -\beta V_{T_1} N\left[d_{1, V_{T_1}/F}(T_2 - T_1)\right] + Fe^{-r(T_2 - T_1)} N\left[d_{2, V_{T_1}/F}(T_2 - T_1)\right] \equiv H^{LS}(V_{T_1}, F, T_2).$$

One therefore partly benefits from his analysis. He has demonstrated that, in presence of liquidation costs, there exists an optimal investment horizon for restructuring debt in default. In other terms, he shows that, when $F > V_{T_1}$, there exists τ such that $H^{LS}(V_{T_1}, F, T_1 + \tau)$ is maximum and such that $H^{LS}(V_{T_1}, F, T_1 + \tau)$ is positive. This is of course valid for $F = F_1 - A$, as long as $F_1 - A > V_{T_1}$. For other values $(A > F_1 - V_{T_1})$, the new debt is no more in distress and, in our structural framework, it is already known that there exists no optimal debt horizon to consider. There is no real sense to consider such environments, because we claim that restructuring is admissible only when an exchanged face value A is constrained to be equal to the amount of new equity received. For $\tau = 0$ creditors can only liquidate the firm meaning that $H(V_{T_1}, F_1, T_1; A, 0) = 0$. At $\tau = \infty$, it is not difficult to show that $H(V_{T_1}, F_1, T_1 + \infty; A, 0) = (\theta - \beta) V_{T_1}$. At the limit, the net gain value may be negative or positive depending on whether θ is smaller or greater than β. For $\theta < \beta$, the net gain function tends to a negative and finite limit value (from above). Because the net gain function is continuous, this implies that the maximum exists and may be found (at worst) at $\tau = 0$. For $\theta > \beta$, the net gain function tends to a positive and finite value which could however be the maximum value possible. Rewriting the net gain function as

$$H(V_{T_1}, F_1, T_2; A, \theta) = \left[D_n(V_{T_1}, F_1 - A, T_2 - T_1) - \beta V_{t_1}\right] + \theta C_n(V_{T_1}, F_1 - A, T_2 - T_1)$$

may help to analyze this. This expression shows that the Longstaff's analysis concerns the first term (in brackets) only. In great generality (i.e. with no constraint on parameters), one cannot ensure that the second (optional) component does not imply a strictly increasing behaviour in T_2. Extensive numerical experiments show that this is not the case (except for very extreme scenario). Constraining parameters may however change things significantly. To understand this, let's follow a two-step approach. Let's first assume that the first constraint (equation (7)) is verified so that the value of the option can be equal to a constant A. The second step is now to find the investment period so that $H(V_{T_1}, F_1, T_2; A, \theta) = \left[D_n(V_{T_1}, F_1 - A, T_2 - T_1) - \beta V_{t_1}\right] + A$ admits a maximum. This problem is very closed to that of Longstaff (1990) except that it adds a constant – insensible to the T_2. Then this does not question the existence of a maximum.