

TOPICS FOR SMALL LECTURES

(1) **The theory of sheaves**

Present this theory as in the theory of étale covers.

REFERENCE: [MK06]

(2) **On the $\bar{\partial}$ -Poincaré lemma = Dolbeault lemma**

The aim is to give a complete proof of the lemma and to recall its consequences for the Dolbeault complex on a complex manifold.

REFERENCE: [GH94, pages 25-27]

(3) **Vector fields on Hirzebruch surfaces**

Let V be a 2-dimensional complex vector space. Let m be an integer. Denote by

$$F_m := \mathbb{P}(\mathcal{O}_{\mathbb{P}(V)} \oplus \mathcal{O}_{\mathbb{P}(V)}(m)) \xrightarrow{\pi} \mathbb{P}(V).$$

Using the relative tangent sequence that computes the tangent bundle TF_m of F_m in terms of $TF_m/\mathbb{P}(V) := \text{Ker}d\pi$ and $\pi^*T\mathbb{P}(V)$, the relative Euler sequence (i.e. the description of $TF_m/\mathbb{P}(V)$ in terms of $\pi^*(\mathcal{O}_{\mathbb{P}(V)} \oplus \mathcal{O}_{\mathbb{P}(V)}(m))$) and the universal quotient bundle $\mathcal{O}_{F_m}(1)$, compute the dimension of the space of holomorphic vector fields on the Hirzebruch surface F_m .

REFERENCE: [Dem, page 281]

(4) **Kodaira-Spencer class**

Give explicit computations of the Kodaira-Spencer class for families of compact complex manifolds. You may choose to present the Čech version or the Dolbeault version.

REFERENCE: [MK06]

(5) **Deformations of Iwasawa manifolds**

The aim is to give an explicit construction of the Kuranishi family of the Iwasawa manifold.

REFERENCE: [Nak75, pages 94-95]

(6) **Period parallelograms and cubic curves**

The theme is to describe the moduli of cubic curves.

REFERENCE: [Muk03, pages 41-50]

(7) **Classical invariants and moduli of hypersurfaces**

The aim is to describe a way of parametrising the set of hypersurfaces of a given degree in a complex projective space.

REFERENCE: [Muk03, pages 167-170]

REFERENCES

- [Dem] Jean-Pierre Demailly. *Complex Analytic and Differential Geometry*. [link to the book](#).
- [GH94] Phillip Griffiths and Joseph Harris. *Principles of algebraic geometry*. Wiley Classics Library. John Wiley & Sons, Inc., New York, 1994. Reprint of the 1978 original, [link](#).
- [MK06] James Morrow and Kunihiko Kodaira. *Complex manifolds*. AMS Chelsea Publishing, Providence, RI, 2006. Reprint of the 1971 edition with errata.
- [Muk03] Shigeru Mukai. *An introduction to invariants and moduli*, volume 81 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2003. Translated from the 1998 and 2000 Japanese editions by W. M. Oxbury.
- [Nak75] Iku Nakamura. Complex parallelisable manifolds and their small deformations. *J. Differential Geometry*, 10:85–112, 1975. [link to the article](#).