

QCM de l'Actuariat Nord Américain

Il s'agit ici de petits exercices très courts, qui doivent être traités en 5 minutes chacun.

Exercice 1. *You are given $\mathbb{P}(A \cup B) = 0.7$ and $\mathbb{P}(A \cup B') = 0.9$. Determine $\mathbb{P}(A)$.*

- 0.2
- 0.3
- 0.4
- 0.6
- 0.8

Exercice 2. *An urn contains 10 balls, 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44. Calculate the number of blue balls in the second urn.*

- 4
- 20
- 24
- 44
- 64

Exercice 3. *An auto insurance company has 10,000 policy holders. Each policyholder is classified as*

1. *young or old;*
2. *male or female; and*
3. *married or single.*

Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males. How many of the company's policyholders are young, female, and single?

- 280
- 423
- 486
- 880
- 896

Exercice 4. *A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease. Determine the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.*

- 11.5%
- 17.3%
- 22.4%
- 32.7%
- 51.4%

Exercise 5. An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto and a homeowners policy will renew at least one of those policies next year. Company records show that 65% of policyholders have an auto policy, 50% policyholders have a homeowners policy, and 15% of policyholders have both an auto and a homeowners policy. Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

- 20%
- 29%
- 41%
- 53%
- 70%

Exercise 6. An insurance company examines its pool of auto insurance customers and gathers the following information:

1. All customers insure at least one car.
2. 70% of the customers insure more than one car.
3. 20% of the customers insure a sports car.
4. Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

- 13%
- 21%
- 24%
- 25%
- 30%

Exercise 7. An insurance company examines its pool of auto insurance customers and gathers the following information:

1. All customers insure at least one car.
2. 64% of the customers insure more than one car.
3. 20% of the customers insure a sports car.
4. Of those customers who insure more than one car, 15% insure a sports car.

What is the probability that a randomly selected customer insures exactly one car, and that car is not a sports car?

- 16%

- 19%
- 26%
- 29%
- 31%

Exercise 8. *An actuary studying the insurance preferences of automobile owners makes the following conclusions:*

1. *An automobile owner is twice as likely to purchase collision coverage as disability coverage.*
2. *The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.*
3. *The probability that an automobile owner purchases both collision and disability coverages is 0.15.*

What is the probability that an automobile owner purchases neither collision nor disability coverage?

- 18%
- 33%
- 48%
- 67%
- 82%

Exercise 9. *In modeling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$, $p_{n+1} = p_n/5$, where p_n represents the probability that the policyholder files n claims during the period. Under this assumption, what is the probability that a policyholder files more than one claim during the period?*

- 4%
- 16%
- 20%
- 80%
- 96%

Exercise 10. *An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims. Calculate the probability that a claim submitted to the insurance company includes operating room charges.*

- 10%
- 20%
- 25%
- 40%
- 80%

Exercise 11. *An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:*

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	0.06	0.08
21-30	0.03	0.15
31-65	0.02	0.49
66-99	0.04	0.28

A randomly selected driver that the company insures has an accident. Calculate the probability that the driver was age 16-20.

- 13%
- 16%
- 19%
- 23%
- 40%

Exercise 12. A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers. Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers. A randomly selected participant from the study died over the five-year period. Calculate the probability that the participant was a heavy smoker.

- 20%
- 25%
- 35%
- 42%
- 57%

Exercise 13. The number of injury claims per month is modeled by a random variable N with

$$\mathbb{P}(N = n) = \frac{1}{(n+1)(n+2)}, \text{ where } n \geq 0.$$

Determine the probability of at least one claim during a particular month, given that there have been at most four claims during that month.

- $1/3$
- $2/5$
- $1/2$
- $3/5$
- $5/6$

Exercise 14. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease. Calculate the probability that a person has the disease given that the test indicates the presence of the disease.

- 32.4%
- 65.7%
- 94.5%
- 95.0%

- 99.5%

Exercise 15. An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims. If the number of claims filed has a Poisson distribution, what is the variance of the number of claims filed?

- $1/\sqrt{3}$
- 1
- $\sqrt{2}$
- 2
- 4

Exercise 16. The loss due to a fire in a commercial building is modeled by a random variable X with density function $f(x) = 0.005(20 - x)\mathbf{1}(x \in]0, 20[)$. Given that a fire loss exceeds 8, what is the probability that it exceeds 16 ?

- 1/25
- 1/9
- 1/8
- 1/3
- 3/7

Exercise 17. An insurance policy pays 100 per day for up to 3 days of hospitalization and 50 per day for each day of hospitalization thereafter. The number of days of hospitalization, X , is a discrete random variable with probability function

$$\mathbb{P}(X = k) = \frac{6 - k}{15} \text{ for } k = 1, 2, 3, 4, 5, \text{ and } 0 \text{ otherwise.}$$

Determine the expected payment for hospitalization under this policy.

- 123
- 210
- 220
- 270
- 367

Exercise 18. An auto insurance company insures an automobile worth 15,000 for one year under a policy with a 1,000 deductible. During the policy year there is a 0.04 chance of partial damage to the car and a 0.02 chance of a total loss of the car. If there is partial damage to the car, the amount X of damage (in thousands) follows a distribution with density function $f(x) = 0.5003e^{-x/2}\mathbf{1}(x \in]0, 15[)$. What is the expected claim payment?

- 320
- 328
- 352
- 380
- 540

Exercise 19. An insurer's annual weather-related loss, X , is a random variable with density function $f(x) = \frac{2.5(200)^{2.5}}{x^{3.5}}$ for $x > 200$, and 0 otherwise. Calculate the difference between the 30th and 70th percentiles of X .

- 35
- 93
- 124
- 231
- 298

Exercise 20. The time to failure of a component in an electronic device has an exponential distribution with a median of four hours. Calculate the probability that the component will work without failing for at least five hours.

- 7%
- 29%
- 38%
- 42%
- 57%

Exercise 21. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f(x, y) = \frac{x + y}{2} \text{ for } 0 < x, y < 2.$$

What is the probability that the device fails during its first hour of operation?

- 12.5%
- 14.1%
- 39.1%
- 62.5%
- 87.5%

Exercise 22. Claims filed under auto insurance policies follow a normal distribution with mean 19,400 and standard deviation 5,000. What is the probability that the average of 25 randomly selected claims exceeds 20,000 ?

- 1%
- 15%
- 27%
- 33%
- 45%

Exercise 23. A city has just added 100 new female recruits to its police force. The city will provide a pension to each new hire who remains with the force until retirement. In addition, if the new hire is married at the time of her retirement, a second pension will be provided for her husband. A consulting actuary makes the following assumptions:

1. Each new recruit has a 0.4 probability of remaining with the police force until retirement.

2. Given that a new recruit reaches retirement with the police force, the probability that she is not married at the time of retirement is 0.25 .
3. The number of pensions that the city will provide on behalf of each new hire is independent of the number of pensions it will provide on behalf of any other new hire.

Determine the probability that the city will provide at most 90 pensions to the 100 new hires and their husbands.

- 60%
- 67%
- 75%
- 93%
- 99%

Exercise 24. Let X denote the size of a surgical claim and let Y denote the size of the associated hospital claim. An actuary is using a model in which $\mathbb{E}(X) = 5$, $\mathbb{E}(X^2) = 27.4$, $\mathbb{E}(Y) = 7$, $\mathbb{E}(Y^2) = 51.4$, and $\text{Var}(X + Y) = 8$. Let $C_1 = X + Y$ denote the size of the combined claims before the application of a 20% surcharge on the hospital portion of the claim, and let C_2 denote the size of the combined claims after the application of that surcharge. Calculate $\text{Cov}(C_1, C_2)$

- 8.80
- 9.60
- 9.76
- 11.52
- 12.32

Exercise 25. The number of workplace injuries, N , occurring in a factory on any given day is Poisson distributed with mean λ . The parameter λ is a random variable that is determined by the level of activity in the factory, and is uniformly distributed on the interval $[0, 3]$. Calculate $\text{Var}(N)$.

- λ
- 2λ
- 0.75
- 1.50
- 2.25

N'hésitez pas à me recontacter pour plus d'exos, si vous n'en avez pas assez...

Calculs de probabilité

Exercise 26. Montrer que pour toute variable positive X telle que $\mathbb{E}(|X|) < \infty$,

$$\mathbb{E}(X) = \int_0^{\infty} \bar{F}(x) dx, \text{ où } \bar{F}(x) = 1 - F(x) = \mathbb{P}(X > x).$$

Exercise 27. Montrer que pour toute variable X d'espérance finie $\mathbb{E}(X) = \int_0^1 F_X^{-1}(p) dp$. En déduire

que $\mathbb{E}([X - k]_+) = \int_{F_X(k)}^1 F_X^{-1}(p) dp - k\bar{F}_X(k)$.

Exercice 28. Soit Θ une variable suivant une loi Gamma de paramètres (α, β) , de densité

$$f(x; \alpha, \beta) = x^{\alpha-1} \frac{\beta^\alpha e^{-\beta x}}{\Gamma(\alpha)} \text{ pour } x > 0.$$

Calculez $\mathbb{E}(\Theta)$ et $\text{Var}(\Theta)$.

Exercice 29. Si X suit une loi de Pareto de paramètres α et θ , donnez la loi de $\log(1 + X/\theta)$.

Exercice 30. Soit Θ une variable suivant une loi Gamma de paramètres (α, β) . On suppose que $N|\Theta$ suit une loi de Poisson de paramètre Θ . Donnez la loi non-conditionnelle de N .

Exercice 31 (suite de l'exercice précédent). Donnez la loi dite a posteriori de Θ sachant N . Montrer qu'il s'agit d'une loi Gamma de paramètres $(\alpha + N, \beta + 1)$.

Exercice 32. Soit N une loi Poisson mélange, de paramètre d'hétérogénéité Θ . Montrer que $\text{Var}(N) > \mathbb{E}(N)$.

Exercice 33. Soient X_1 et X_2 deux lois binomiales négatives indépendantes, de paramètres respectifs (α_1, p) et (α_2, p) , respectivement. Montrer que $X_1 + X_2$ suit une loi binomiale négative dont on donnera les paramètres.

Exercice 34. Soient X_1, \dots une suite de variables indépendantes de loi uniforme sur $[0, 1]$.

- Donner la fonction de répartition de $X_1 + X_2$.
- Montrer que $\mathbb{P}(X_1 + \dots + X_n \leq x) = \frac{1}{n!} \sum_{k=0}^{n-1} (-1)^k \binom{n}{k} [x - k]_+^n$ pour $0 \leq x \leq n$, où $[x - k]_+$ vaut $x - k$ si $x > k$, et 0 sinon.
- En déduire la densité de la somme de n terme, ainsi que la densité de la moyenne.

Exercice 35. Soient X_1, \dots une suite de variables indépendantes de loi exponentielle de paramètre α . On pose $S_n = X_1 + \dots + X_n$.

- Montrer que les fonctions de répartition de S_n et de S_{n+1} sont liées par une relation de la forme

$$F_{S_{n+1}}(x) = \int_0^\infty F_{S_n}(x - t) dF(x).$$

- En déduire la fonction de répartition de S_n .

complément sur les lois usuelles

Pour les lois de comptage

Loi de probabilité	Support	Probabilités
$\mathcal{D}Uni(m)$	$\{0, 1, \dots, m\}$	$\frac{1}{m+1}$
$\mathcal{B}er(q)$	$\{0, 1\}$	$q^k(1-q)^{1-k}$
$\mathcal{B}in(m, q)$	$\{0, 1, \dots, m\}$	$\binom{m}{k} q^k(1-q)^{m-k}$ où $\binom{m}{k} = \frac{m!}{k!(m-k)!}$
$\mathcal{G}eo(q)$	\mathbb{N}	$q(1-q)^k$
$\mathcal{N}Bin(\alpha, q)$	\mathbb{N}	$\binom{\alpha+k-1}{k} q^k(1-q)^{\alpha-k}$ où $\binom{\alpha+k-1}{k} = \frac{\Gamma(\alpha+k)}{\Gamma(k+1)\Gamma(\alpha)} = \frac{\Gamma(\alpha+k)}{k!\Gamma(\alpha)}$
$\mathcal{P}oi(\lambda)$	\mathbb{N}	$\exp(-\lambda) \frac{\lambda^k}{k!}$

Pour les lois continues

Loi de probabilité	Support	Densité
$Uni(a, b)$	$[a, b]$	$\frac{1}{b-a}$
$\mathcal{B}et(\alpha, \beta)$	$[0, 1]$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$
$\mathcal{N}or(\mu, \sigma^2)$	\mathbb{R}	$\frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{1}{2\sigma^2}(x-\mu)^2)$
$\mathcal{LN}or(\mu, \sigma^2)$	\mathbb{R}^+	$\frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2\right)$
$\mathcal{E}xp(\theta)$	\mathbb{R}^+	$\theta \exp(-\theta x)$
$\mathcal{G}am(\alpha, \tau)$	\mathbb{R}^+	$\frac{x^{\alpha-1}\tau^\alpha \exp(-x\tau)}{\Gamma(\alpha)}$
$\mathcal{P}ar(\alpha, \theta)$	\mathbb{R}^+	$\frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}}$

Quelques calculs de moyennes

Loi	Moyenne	Loi	Moyenne
$\mathcal{D}Uni(n)$	$\frac{n}{2}$	$\mathcal{N}or(\mu, \sigma^2)$	μ
$\mathcal{B}er(q)$	q	$\mathcal{LN}or(\mu, \sigma^2)$	$\exp(\mu + \frac{\sigma^2}{2})$
$\mathcal{B}in(m, q)$	mq	$\mathcal{E}xp(\theta)$	$1/\theta$
$\mathcal{G}eo(q)$	$\frac{1-q}{q}$	$\mathcal{G}am(\alpha, \tau)$	$\frac{\alpha}{\tau}$
$\mathcal{N}Bin(\alpha, q)$	$\frac{\alpha(1-q)}{q}$	$\mathcal{P}ar(\alpha, \theta)$	$\frac{\theta}{\alpha-1}$ si $\alpha > 1$
$\mathcal{P}oi(\lambda)$	λ	$\mathcal{B}et(\alpha, \beta)$	$\frac{\alpha}{\alpha+\beta}$
		$Uni(a, b)$	$\frac{a+b}{2}$

Quelques calculs de variances

Loi de probabilité	Variance
$\mathcal{D}Uni(n)$	$\frac{1}{n+1} \sum_{j=0}^n j^2 - \frac{n^2}{4}$
$\mathcal{B}er(q)$	$q(1-q)$
$\mathcal{B}in(m, q)$	$mq(1-q)$
$\mathcal{G}eo(q)$	$\frac{1-q}{q^2}$
$\mathcal{N}Bin(\alpha, q)$	$\frac{\alpha(1-q)}{q^2}$
$\mathcal{P}oi(\lambda)$	λ

Loi de probabilité	Variance
$\mathcal{N}or(\mu, \sigma^2)$	σ^2
$\mathcal{LN}or(\mu, \sigma^2)$	$\exp(2\mu + \sigma^2)(\exp(\sigma^2) - 1)$
$\mathcal{E}xp(\theta)$	$\frac{1}{\theta^2}$
$\mathcal{G}am(\alpha, \tau)$	$\frac{\alpha}{\tau^2}$
$\mathcal{P}ar(\alpha, \theta)$	$\frac{\alpha\theta^2}{(\alpha-2)(\alpha-1)^2}$ si $\alpha > 2$
$\mathcal{B}et(\alpha, \beta)$	$\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$
$Uni(a, b)$	$\frac{(b-a)^2}{12}$