Differential equations over a 1-dimensional affinoid. p-adic deformation of differential equations.

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Rennes, 5-10 July 2009.

3 Applications/Examples:

3.1 (ϕ, Γ)-modules

- From the action of ϕ, Berger construct a differential equation with singularities over the Robba ring.
  - The representation is of do Pham type if and only if the differential equation can be "desingularized".

Proposition:
- There exists a subgroup of I acting infinitesimally and in a non degenerate way on the Robba ring.
- The action of a subgroup of I (after desingularization) can be recovered from the differential equation by deformation.
- In other words the action of a subgroup of I coincides with the action deduced from the differential equation.

3.2 difference equations

- If f(T + ϕ(T)) = f(QT) then one speaks about p-difference equations.
- If f(T + ϕ(T)) = f(QT + h) then one speaks about finite difference equations.
  - More generally we consider the case ε ϕ(QT) = f(QT + h). We consider equations of the type: Y(qT + h) = A(T) Y(T).

FACT: we have a stronger result because there exists a twisted Taylor formula expressing the Taylor solution of the above equation.

- Conclusion: form the ε ϕ modules we can recover the cocycle χ and hence also the differential equation.

Theorem: If χ is not a root of unity, then the ε ϕ deformation functor is an equivalence.

3.3 Morita’s p-adic Gamma function and L-functions of Kubota-Leopoldt

- Difference equation: Γ ((x + 1)) = {−Γ(x) if |x| < 1 − ε ϕ(x) if |x| = 1}.
- Then one has a concrete difference equation: Γ |x + 1| = −Γ(x + 1) Γ(T + 2) + Γ(T + 1).
- By inverse deformation one has a dif. eq. Γ(T + 1) = f(G(T) T).

Theorem: The dif. eq. is defined on the open disk D *(0, i).
- Its radius is known.
- Its radius is strictly related to the Newton polygon of aϕT.
- Morita (1959) - Diamond (1977): log(Γ) = −∑ n0 ε ϕ n. Then for all n0 = ε values at positive integer of a certain L-function.

BOUND: on the valuation n0 via the differential equation.

RECURSIVE RELATION on the n0 via the differential equation.